Conductance-based dendrites perform reliability-weighted opinion pooling

Jakob Jordan,

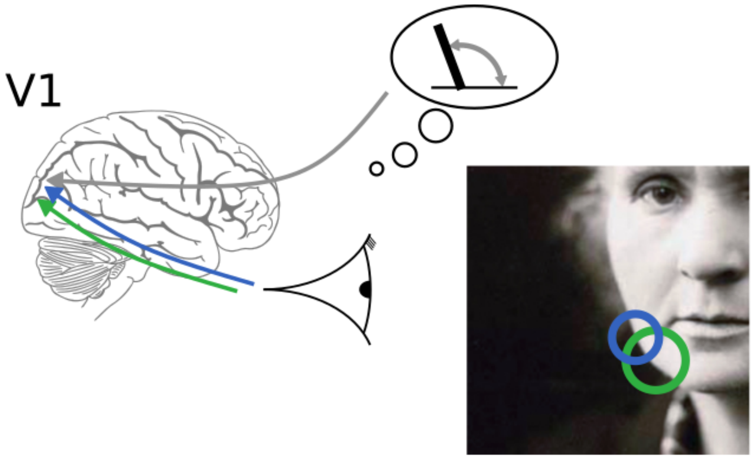
João Sacramento, Mihai A. Petrovici & Walter Senn

Department of Physiology, University of Bern, Switzerland

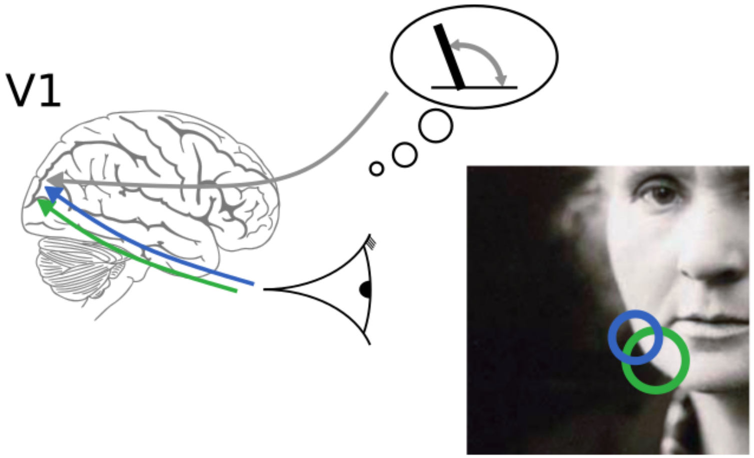
Kirchhoff-Institute for Physics, Heidelberg University, Germany

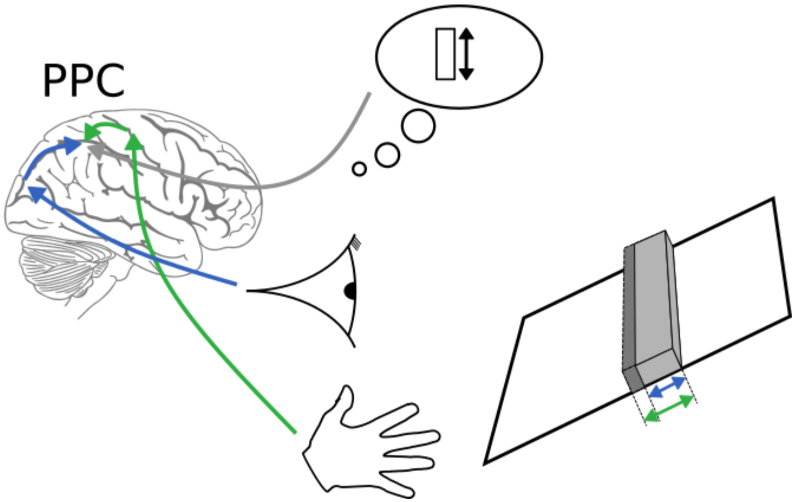
Institute of Neuroinformatics, UZH / ETH Zurich, Switzerland

Cue integration is a fundamental computational principle of cortex

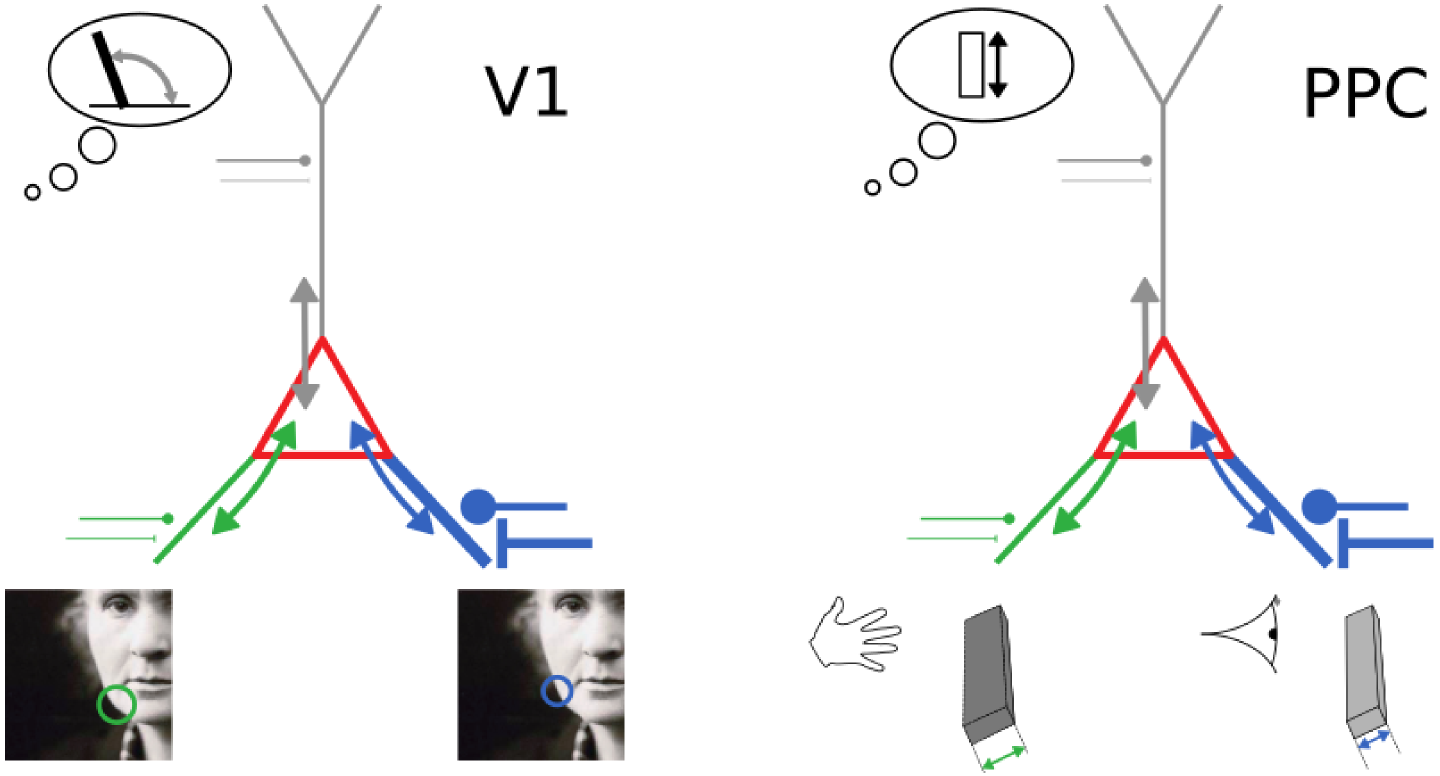


Cue integration is a fundamental computational principle of cortex





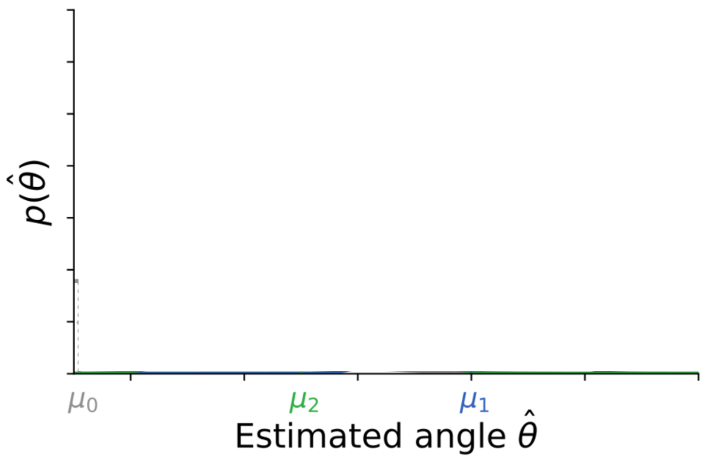
Cue integration is a fundamental computational principle of cortex



Neurons with *conductance-based synapses* naturally implement probabilistic cue integration

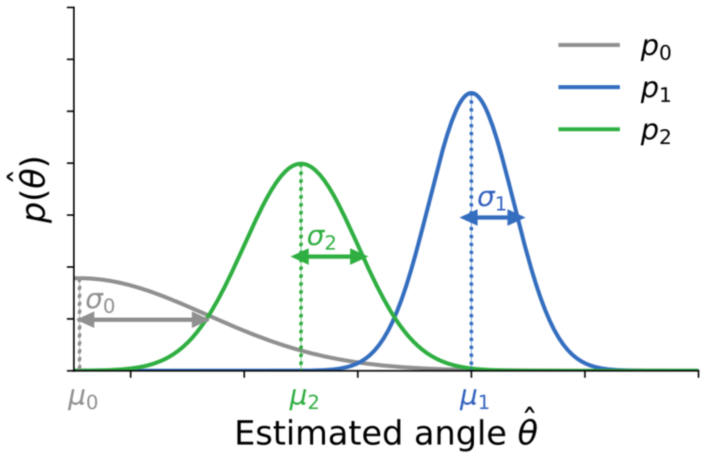
An observation

Bayes-optimal inference



An observation

Bayes-optimal inference



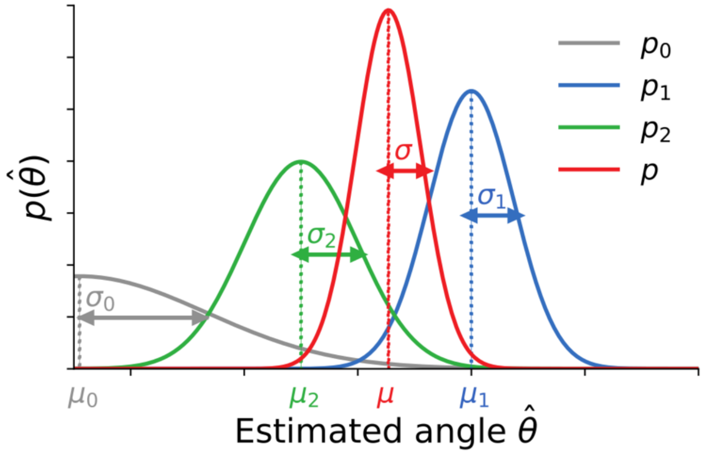
An observation

Bayes-optimal inference

$$\text{mean } \mu$$
$$\text{precision } 1/\sigma^2$$

$$\mu = \frac{\frac{\mu_0}{\sigma_0^2} + \frac{\mu_1}{\sigma_1^2} + \frac{\mu_2}{\sigma_2^2}}{\frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}}$$

$$\frac{1}{\sigma^2} = \frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$$



An observation

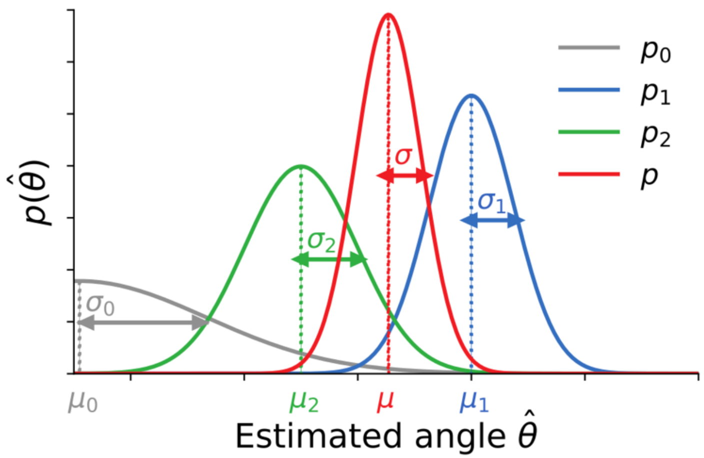
Bayes-optimal inference

mean μ

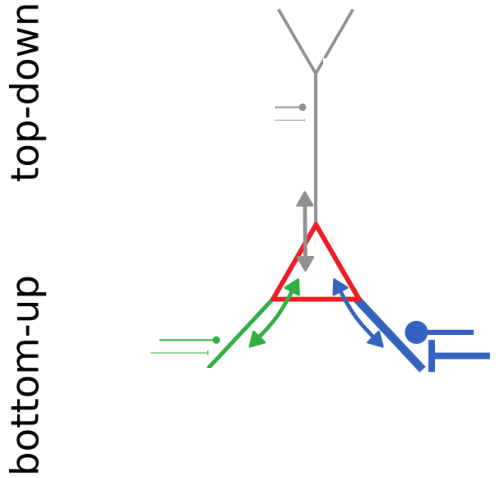
precision $1/\sigma^2$

$$\mu = \frac{\frac{\mu_0}{\sigma_0^2} + \frac{\mu_1}{\sigma_1^2} + \frac{\mu_2}{\sigma_2^2}}{\frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}}$$

$$\frac{1}{\sigma^2} = \frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$$



Bidirectional voltage dynamics



An observation

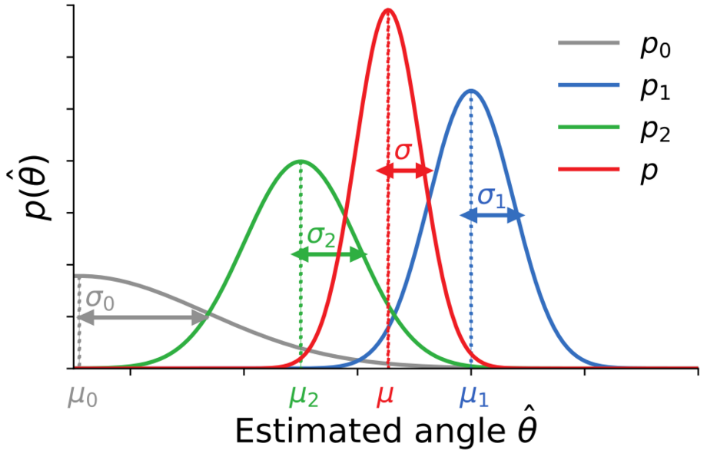
Bayes-optimal inference

mean μ

precision $1/\sigma^2$

$$\mu = \frac{\frac{\mu_0}{\sigma_0^2} + \frac{\mu_1}{\sigma_1^2} + \frac{\mu_2}{\sigma_2^2}}{\frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}}$$

$$\frac{1}{\sigma^2} = \frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$$



Bidirectional voltage dynamics

membrane pot. $\bar{E}_\mathrm{s}$

membrane cond. $\bar{g}_\mathrm{s}$

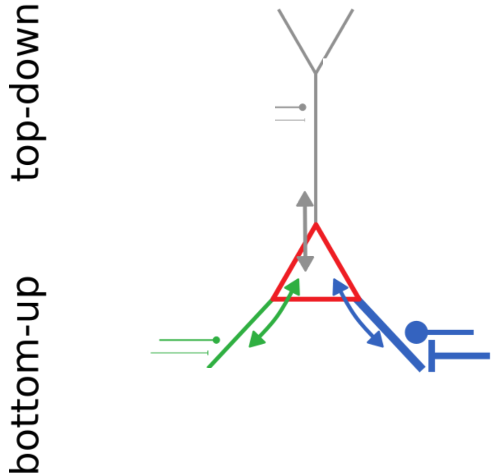
$$\bar{E}_\mathrm{s} = \frac{g_0 E_0 + g_1 E_1 + g_2 E_2}{g_0 + g_1 + g_2}$$

$$\bar{g}_\mathrm{s} = g_0 + g_1 + g_2$$

$$g_0 = g_0^\mathrm{L}$$

$$g_1 = g_1^\mathrm{E} + g_2^\mathrm{I}$$

$$g_2 = g_2^\mathrm{E} + g_2^\mathrm{I}$$



An observation

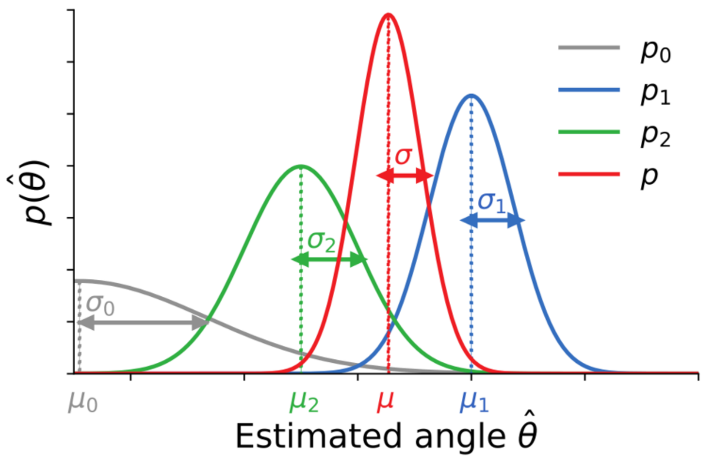
Bayes-optimal inference

mean μ

precision $1/\sigma^2$

$$\mu = \frac{\frac{\mu_0}{\sigma_0^2} + \frac{\mu_1}{\sigma_1^2} + \frac{\mu_2}{\sigma_2^2}}{\frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}}$$

$$\frac{1}{\sigma^2} = \frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$$



Bidirectional voltage dynamics

membrane pot. $\bar{E}_\mathrm{s}$

membrane cond. $\bar{g}_\mathrm{s}$

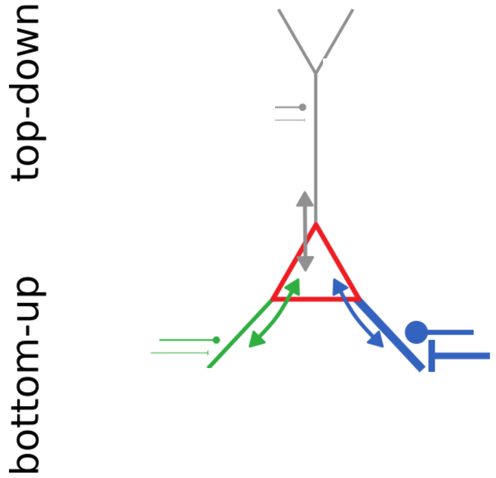
$$\bar{E}_\mathrm{s} = \frac{g_0 E_0 + g_1 E_1 + g_2 E_2}{g_0 + g_1 + g_2}$$

$$\bar{g}_\mathrm{s} = g_0 + g_1 + g_2$$

$g_0 = g_0^\mathrm{L}$

$g_1 = g_1^\mathrm{E} + g_2^\mathrm{I}$

$g_2 = g_2^\mathrm{E} + g_2^\mathrm{I}$



opinion

$\mu \leftrightarrow \bar{E}_\mathrm{s}$

reliability

$1/\sigma^2 \leftrightarrow \bar{g}_\mathrm{s}$

An observation

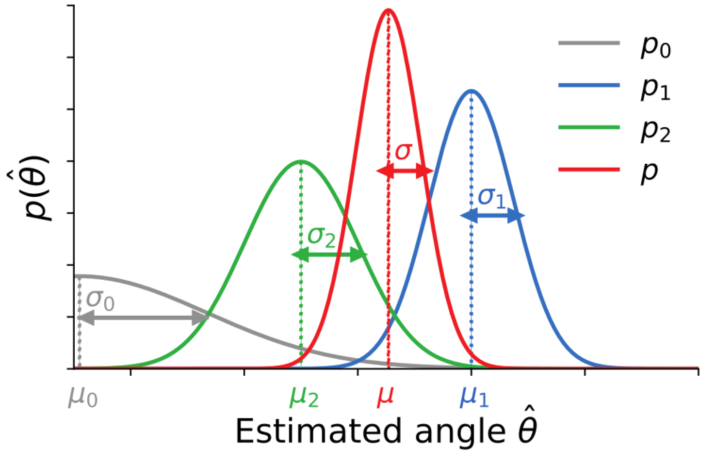
Bayes-optimal inference

mean μ

precision $1/\sigma^2$

$$\mu = \frac{\frac{\mu_0}{\sigma_0^2} + \frac{\mu_1}{\sigma_1^2} + \frac{\mu_2}{\sigma_2^2}}{\frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}}$$

$$\frac{1}{\sigma^2} = \frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$$



Bidirectional voltage dynamics

membrane pot. $\bar{E}_\mathrm{s}$

membrane cond. $\bar{g}_\mathrm{s}$

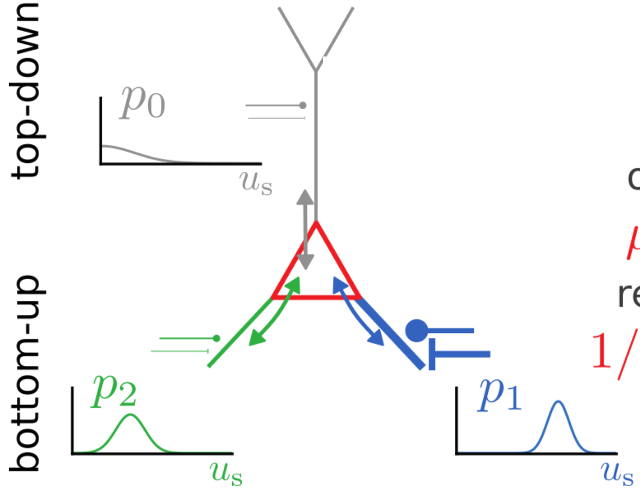
$$\bar{E}_\mathrm{s} = \frac{g_0 E_0 + g_1 E_1 + g_2 E_2}{g_0 + g_1 + g_2}$$

$$\bar{g}_\mathrm{s} = g_0 + g_1 + g_2$$

$g_0 = g_0^\mathrm{L}$

$g_1 = g_1^\mathrm{E} + g_2^\mathrm{I}$

$g_2 = g_2^\mathrm{E} + g_2^\mathrm{I}$



opinion

$\mu \leftrightarrow \bar{E}_\mathrm{s}$

reliability

$1/\sigma^2 \leftrightarrow \bar{g}_\mathrm{s}$

An observation

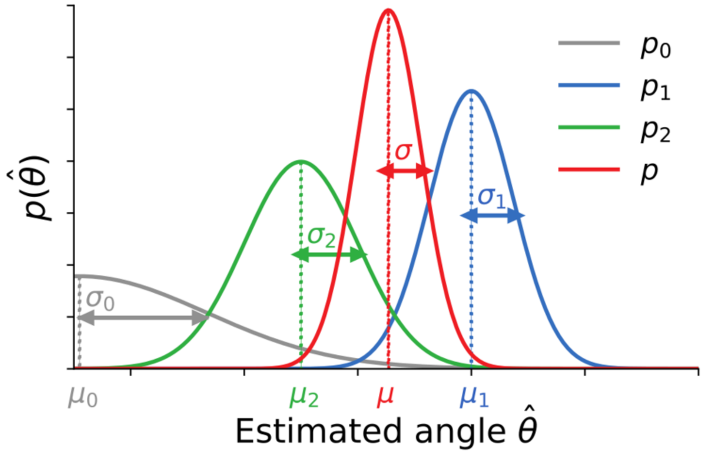
Bayes-optimal inference

mean μ

precision $1/\sigma^2$

$$\mu = \frac{\frac{\mu_0}{\sigma_0^2} + \frac{\mu_1}{\sigma_1^2} + \frac{\mu_2}{\sigma_2^2}}{\frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}}$$

$$\frac{1}{\sigma^2} = \frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$$



Bidirectional voltage dynamics

membrane pot. $\bar{E}_\mathrm{s}$

membrane cond. $\bar{g}_\mathrm{s}$

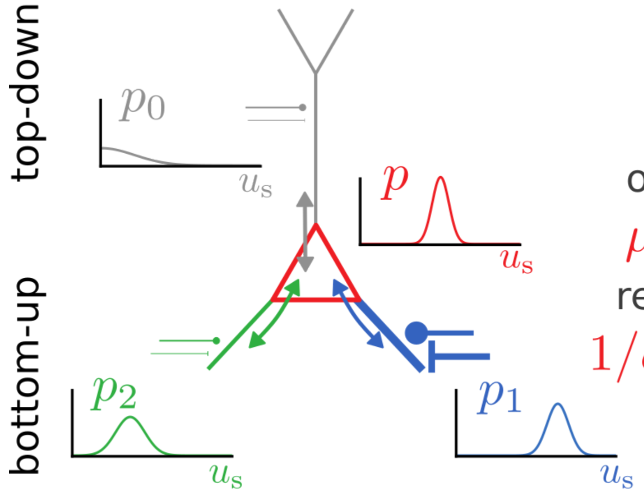
$$\bar{E}_\mathrm{s} = \frac{g_0 E_0 + g_1 E_1 + g_2 E_2}{g_0 + g_1 + g_2}$$

$$\bar{g}_\mathrm{s} = g_0 + g_1 + g_2$$

$$g_0 = g_0^\mathrm{L}$$

$$g_1 = g_1^\mathrm{E} + g_2^\mathrm{I}$$

$$g_2 = g_2^\mathrm{E} + g_2^\mathrm{I}$$



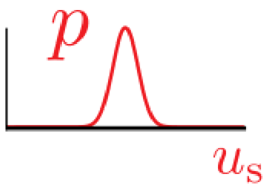
opinion

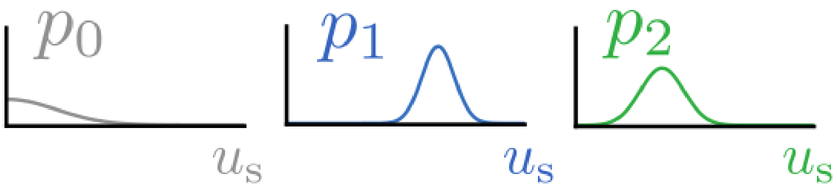
$\mu \leftrightarrow \bar{E}_\mathrm{s}$

reliability

$1/\sigma^2 \leftrightarrow \bar{g}_\mathrm{s}$

Membrane potential dynamics as noisy gradient ascent





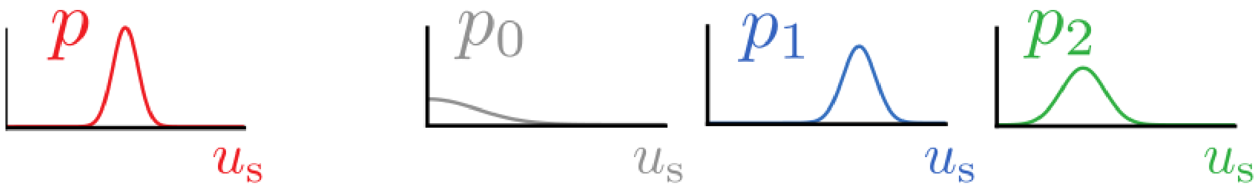
opinion

$\mu \leftrightarrow \bar{E}_\mathrm{s}$

reliability

$1/\sigma^2 \leftrightarrow \bar{g}_\mathrm{s}$

Membrane potential dynamics as noisy gradient ascent



opinion

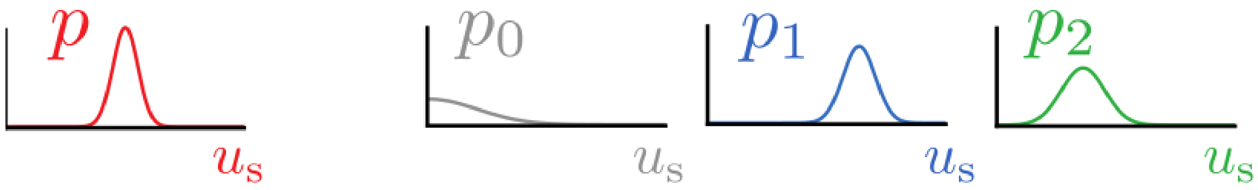
$\mu \leftrightarrow \bar{E}_s$

reliability

$1/\sigma^2 \leftrightarrow \bar{g}_s$

$$p \propto p_0 \, p_1 p_2 \cdots$$

Membrane potential dynamics as noisy gradient ascent



opinion

$\mu \leftrightarrow \bar{E}_s$

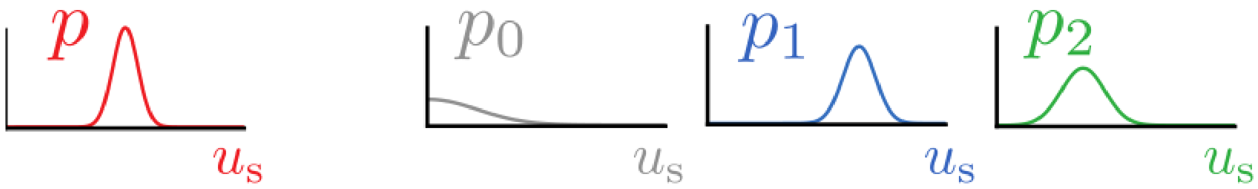
reliability

$1/\sigma^2 \leftrightarrow \bar{g}_s$

$$p \propto p_0 \, p_1 p_2 \cdots$$

$$\begin{aligned} p(u_s | W, r) &= \frac{1}{Z'} \prod_{d=0}^{D} p_d(u_s | W_d, r) \\ &= \frac{1}{Z} e^{-\frac{\bar{g}_s}{2}\left(u_s - \bar{E}_s\right)^2} \end{aligned}$$

Membrane potential dynamics as noisy gradient ascent



opinion

$\mu \leftrightarrow \bar{E}_\mathrm{s}$

reliability

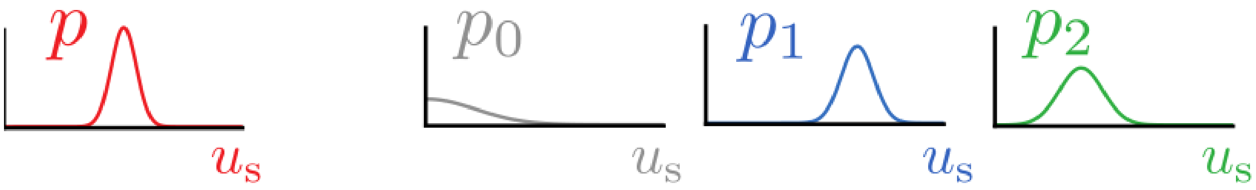
$1/\sigma^2 \leftrightarrow \bar{g}_\mathrm{s}$

$$p \propto p_0 \, p_1 p_2 \cdots$$

$$\begin{aligned} p(u_\mathrm{s}|W,r) &= \frac{1}{Z'} \prod_{d=0}^{D} p_d(u_\mathrm{s}|W_d,r) \\ &= \frac{1}{Z} e^{-\frac{\bar{g}_\mathrm{s}}{2}\left(u_\mathrm{s}-\bar{E}_\mathrm{s}\right)^2} \end{aligned}$$

$$\begin{aligned} C\dot{u}_\mathrm{s} &= \frac{\partial}{\partial u_\mathrm{s}} \log p(u_\mathrm{s}|W,r) + \xi \\ &= \sum_{d=0}^{D} \left(g_d^\mathrm{L}(E^\mathrm{L} - u_\mathrm{s}) + g_d^\mathrm{E}(E^\mathrm{E} - u_\mathrm{s}) + g_d^\mathrm{I}(E^\mathrm{I} - u_\mathrm{s})\right) + \xi \end{aligned}$$

Membrane potential dynamics as noisy gradient ascent



opinion
$\mu \leftrightarrow \bar{E}_\mathrm{s}$
reliability
$1/\sigma^2 \leftrightarrow \bar{g}_\mathrm{s}$

$$p \propto p_0 \, p_1 p_2 \cdots$$

$$\begin{aligned} p(u_\mathrm{s}|W, r) &= \frac{1}{Z'} \prod_{d=0}^{D} p_d(u_\mathrm{s}|W_d, r) \\ &= \frac{1}{Z} e^{-\frac{\bar{g}_\mathrm{s}}{2}\left(u_\mathrm{s} - \bar{E}_\mathrm{s}\right)^2} \end{aligned}$$

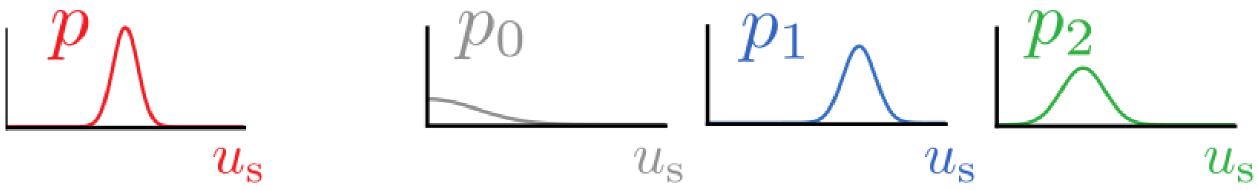
$$\begin{aligned} C\dot{u}_\mathrm{s} &= \frac{\partial}{\partial u_\mathrm{s}} \log p(u_\mathrm{s}|W, r) + \xi \\ &= \sum_{d=0}^{D} \left(g_d^\mathrm{L}(E^\mathrm{L} - u_\mathrm{s}) + g_d^\mathrm{E}(E^\mathrm{E} - u_\mathrm{s}) + g_d^\mathrm{I}(E^\mathrm{I} - u_\mathrm{s}) \right) + \xi \end{aligned}$$

$$\mathbb{E}[u_\mathrm{s}] = \bar{E}_\mathrm{s}$$

Average membrane potentials

== reliability-weighted opinions

Membrane potential dynamics as noisy gradient ascent



opinion

$\mu \leftrightarrow \bar{E}_\mathrm{s}$

reliability

$1/\sigma^2 \leftrightarrow \bar{g}_\mathrm{s}$

$$p \propto p_0 \, p_1 p_2 \cdots$$

$$\begin{aligned} p(u_\mathrm{s}|W,r) &= \frac{1}{Z'} \prod_{d=0}^{D} p_d(u_\mathrm{s}|W_d,r) \\ &= \frac{1}{Z} e^{-\frac{\bar{g}_\mathrm{s}}{2}\left(u_\mathrm{s}-\bar{E}_\mathrm{s}\right)^2} \end{aligned}$$

$$\begin{aligned} C\dot{u}_\mathrm{s} &= \frac{\partial}{\partial u_\mathrm{s}} \log p(u_\mathrm{s}|W,r) + \xi \\ &= \sum_{d=0}^{D} \left(g_d^\mathrm{L}(E^\mathrm{L} - u_\mathrm{s}) + g_d^\mathrm{E}(E^\mathrm{E} - u_\mathrm{s}) + g_d^\mathrm{I}(E^\mathrm{I} - u_\mathrm{s}) \right) + \xi \end{aligned}$$

$$\mathbb{E}[u_\mathrm{s}] = \bar{E}_\mathrm{s}$$

Average membrane potentials

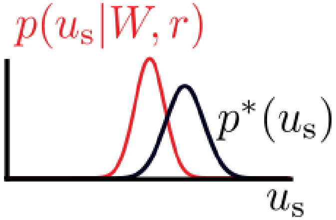
== reliability-weighted opinions

$$\mathrm{Var}[u_\mathrm{s}] = \frac{1}{\bar{g}_\mathrm{s}}$$

Membrane potential variance

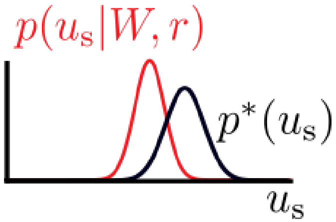
== 1/total reliability

Stochastic-gradient-ascent-based synaptic plasticity



Stochastic-gradient-ascent-based synaptic plasticity

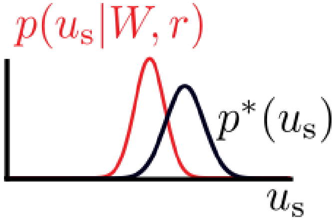
u_s^*: sample from target distribution $p^*(u_s)$



$$\dot{W}_d^{E/I} \propto \frac{\partial}{\partial W_d^{E/I}} \log p(u_s^* | W, r)$$

Stochastic-gradient-ascent-based synaptic plasticity

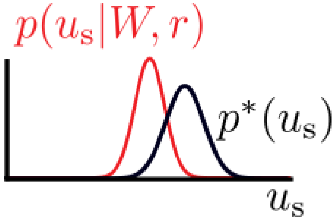
u_s^*: sample from target distribution $p^*(u_\mathrm{s})$



$$\dot{W}_d^{\mathrm{E/I}} \propto \frac{\partial}{\partial W_d^{\mathrm{E/I}}} \log p(u_\mathrm{s}^*|W,r)$$
$$\propto [\, \Delta\mu^{\mathrm{E/I}} + \Delta\sigma^2 \,]\, r$$

Stochastic-gradient-ascent-based synaptic plasticity

u_{s}^{*}: sample from target distribution $p^{*}(u_{\mathrm{s}})$



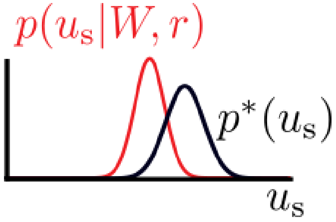
$$\dot{W}_{d}^{\mathrm{E/I}} \propto \frac{\partial}{\partial W_{d}^{\mathrm{E/I}}} \log p(u_{\mathrm{s}}^{*}|W, r)$$

$$\propto [\, \Delta\mu^{\mathrm{E/I}} + \Delta\sigma^{2} \,]\, r$$

$$\Delta\mu^{\mathrm{E/I}} \propto (u_{\mathrm{s}}^{*} - \bar{E}_{\mathrm{s}}) \left(E^{\mathrm{E/I}} - \bar{E}_{\mathrm{s}}\right)$$

Stochastic-gradient-ascent-based synaptic plasticity

u_s^*: sample from target distribution $p^*(u_\mathrm{s})$



$$\dot{W}_d^{\mathrm{E/I}} \propto \frac{\partial}{\partial W_d^{\mathrm{E/I}}} \log p(u_\mathrm{s}^* | W, r)$$

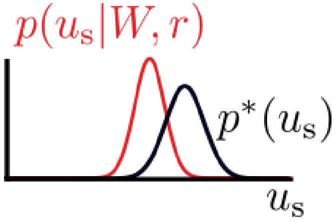
$$\propto [\ \Delta\mu^{\mathrm{E/I}} + \Delta\sigma^2\ ]\, r$$

$$\Delta\mu^{\mathrm{E/I}} \propto (u_\mathrm{s}^* - \bar{E}_\mathrm{s}) \left(E^{\mathrm{E/I}} - \bar{E}_\mathrm{s}\right)$$

$$\Delta\sigma^2 \propto \frac{1}{2} \left(\frac{1}{\bar{g}_\mathrm{s}} - (u_\mathrm{s}^* - \bar{E}_\mathrm{s})^2 \right)$$

Stochastic-gradient-ascent-based synaptic plasticity

u_s^*: sample from target distribution $p^*(u_s)$



$$\dot{W}_d^{E/I} \propto \frac{\partial}{\partial W_d^{E/I}} \log p(u_s^* | W, r)$$

$$\propto [\ \Delta\mu^{E/I} + \Delta\sigma^2\ ]\, r$$

$$\Delta\mu^{E/I} \propto (u_s^* - \bar{E}_s)\left(E^{E/I} - \bar{E}_s\right)$$

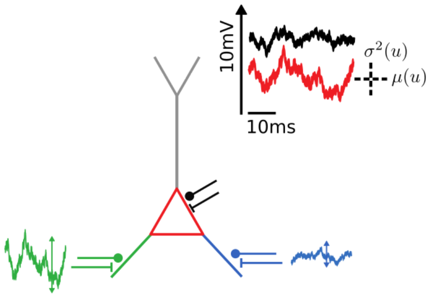
$$\Delta\sigma^2 \propto \frac{1}{2}\left(\frac{1}{\bar{g}_s} - (u_s^* - \bar{E}_s)^2\right)$$

Synaptic plasticity modifies excitatory/inhibitory synapses

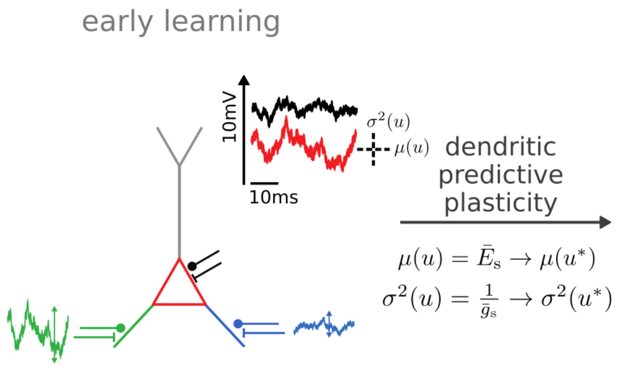
- in approx. opposite directions to match the mean
- in identical directions to match the variance

Synaptic plasticity performs error-correction and reliability matching

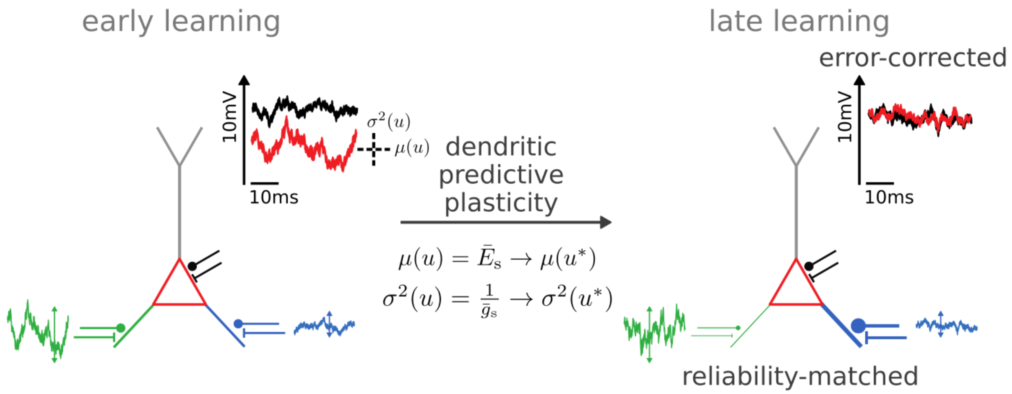
early learning



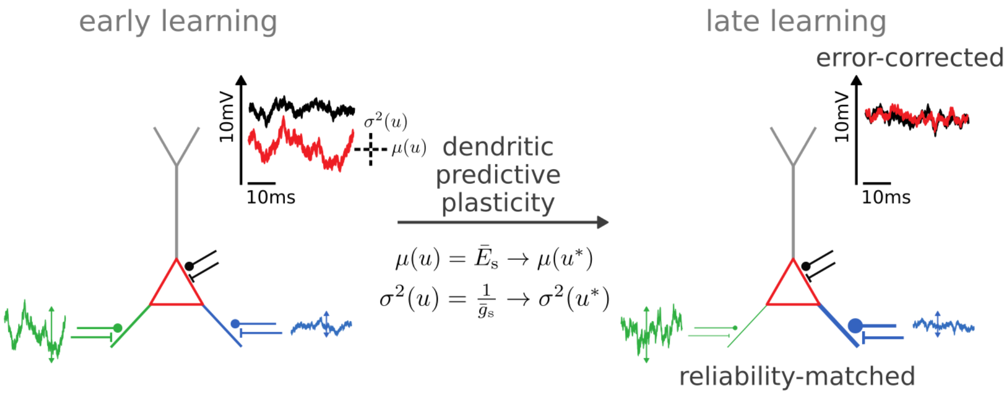
Synaptic plasticity performs error-correction and reliability matching

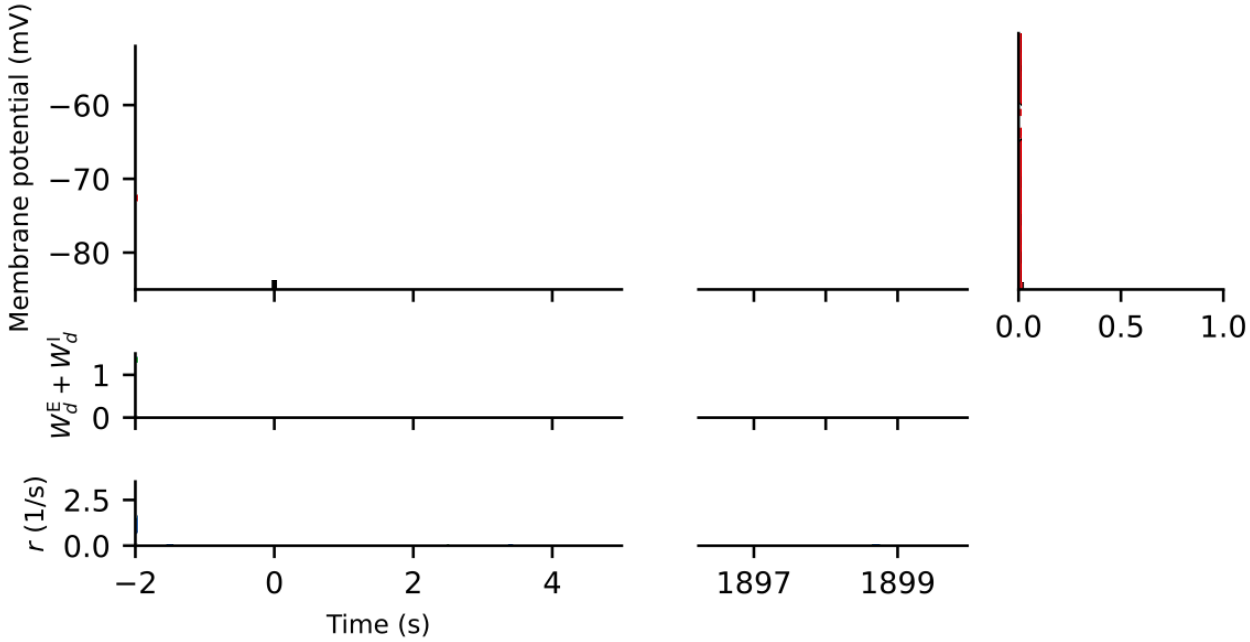


Synaptic plasticity performs error-correction and reliability matching

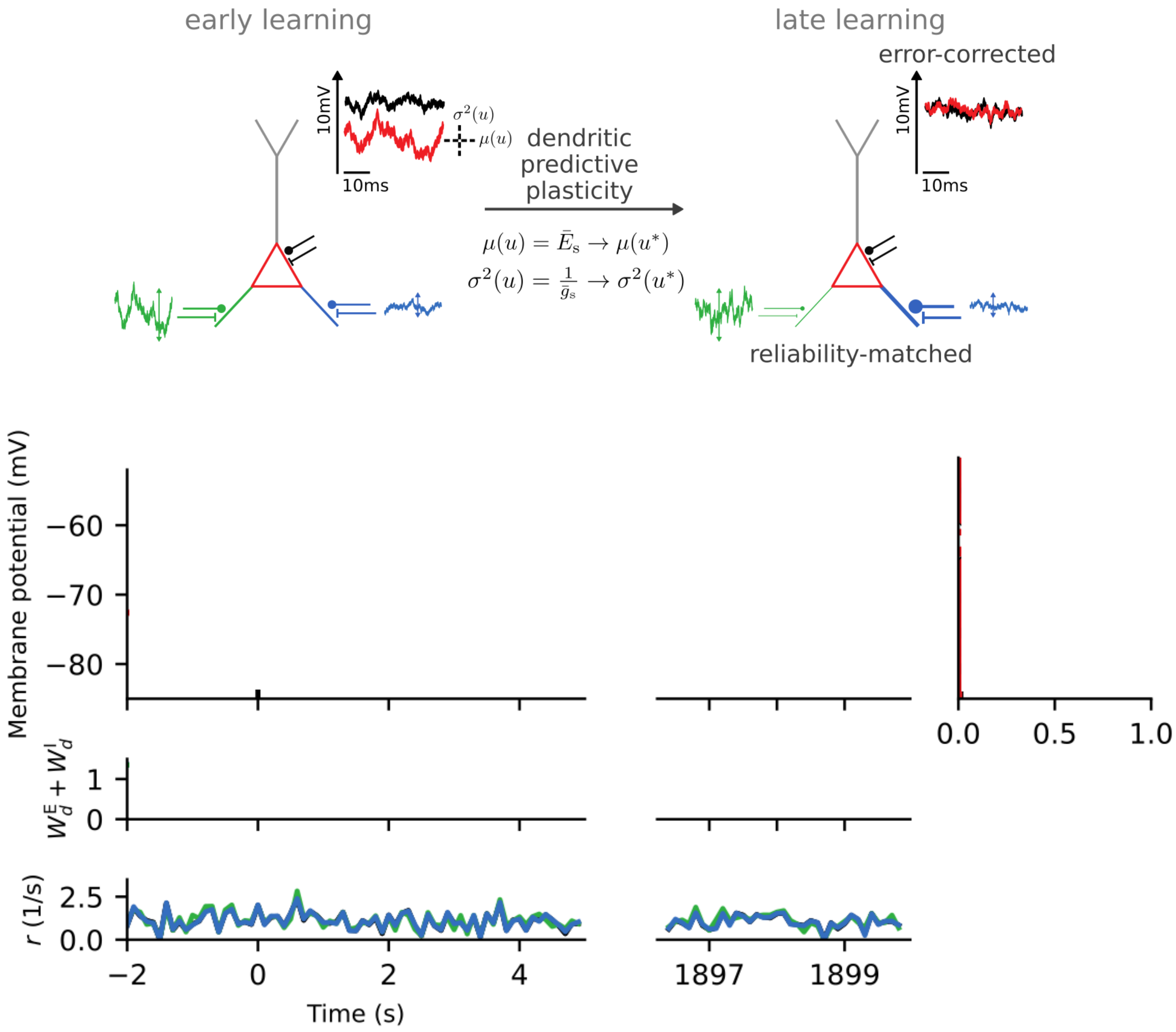


Synaptic plasticity performs error-correction and reliability matching

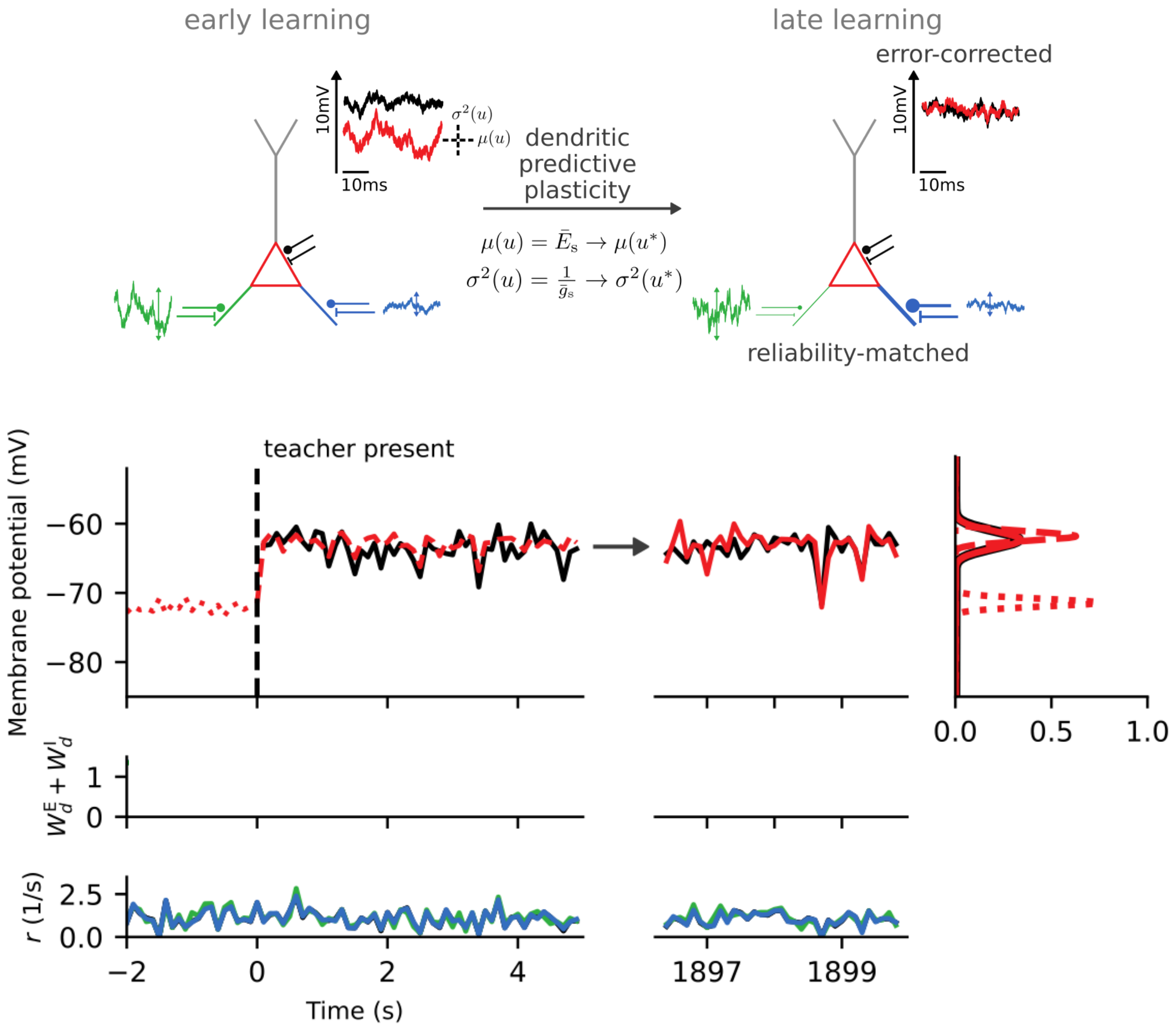




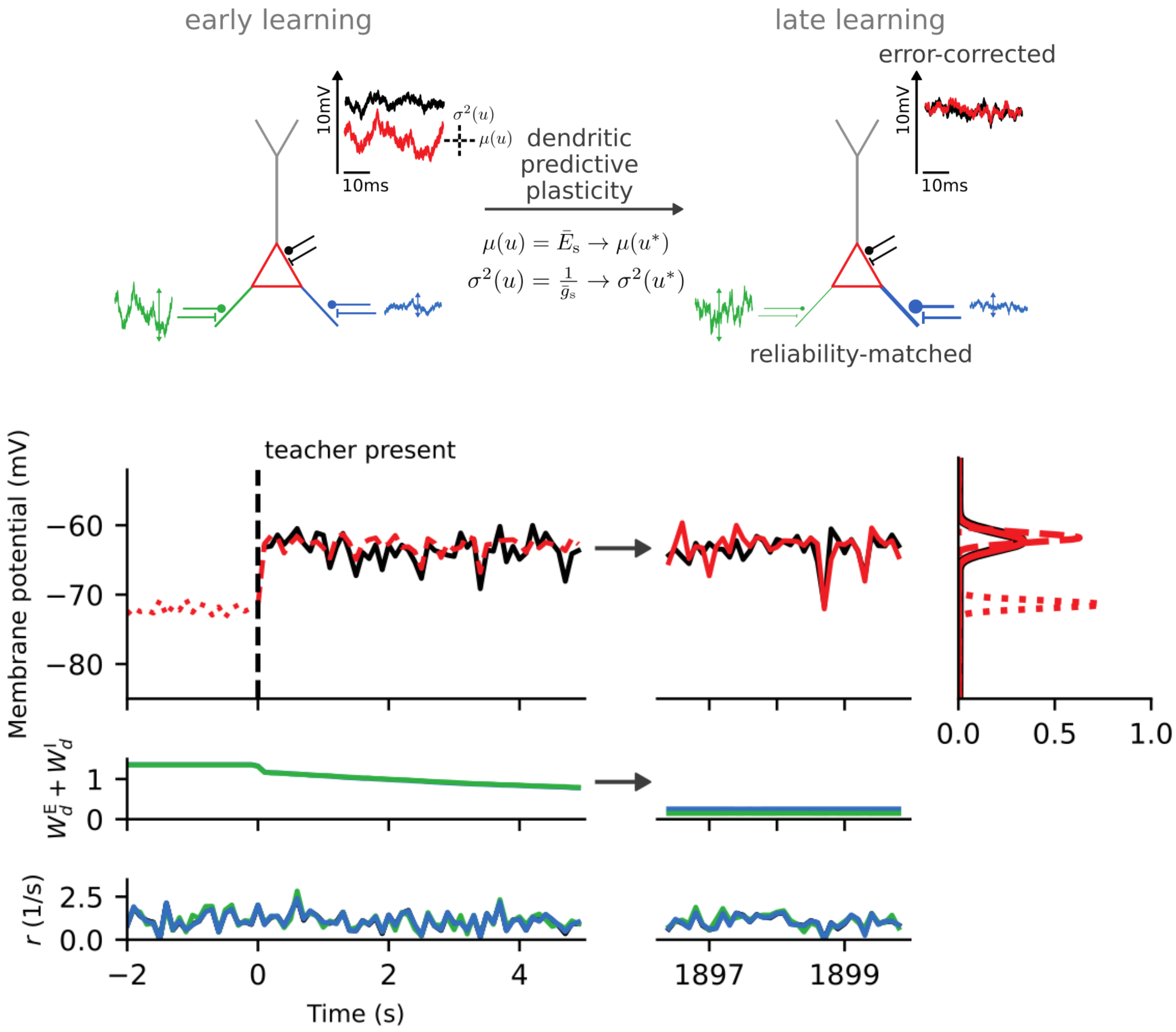
Synaptic plasticity performs error-correction and reliability matching



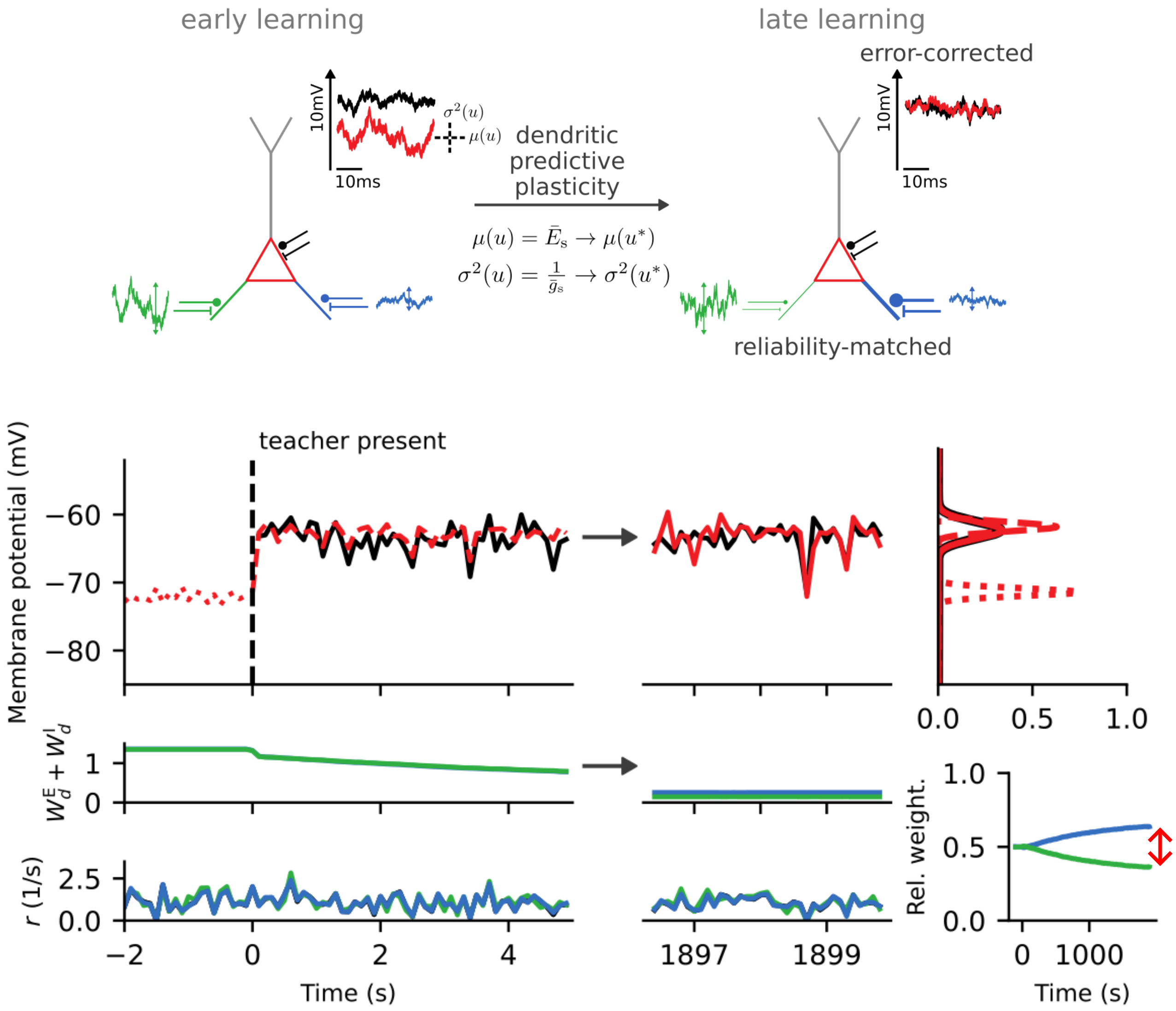
Synaptic plasticity performs error-correction and reliability matching



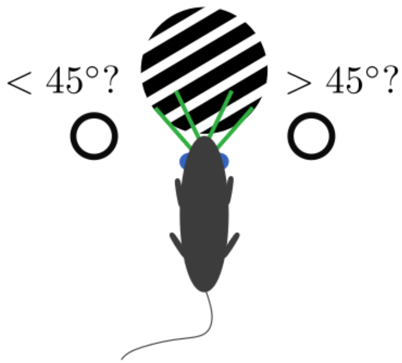
Synaptic plasticity performs error-correction and reliability matching



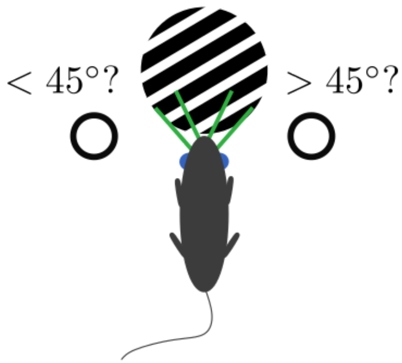
Synaptic plasticity performs error-correction and reliability matching

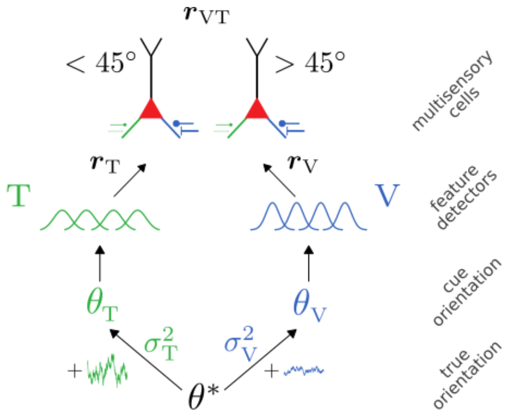


Learning Bayes-optimal inference of orientations from multimodal stimuli

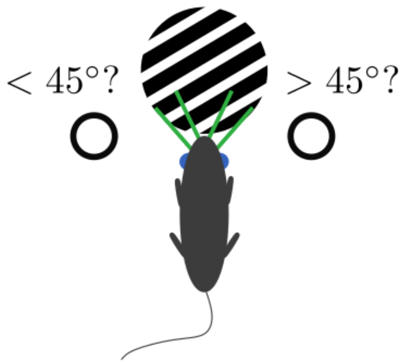


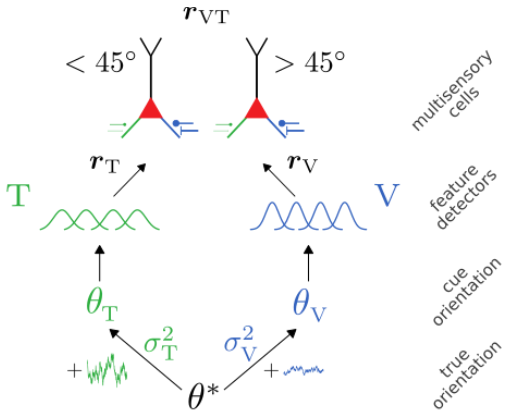
Learning Bayes-optimal inference of orientations from multimodal stimuli

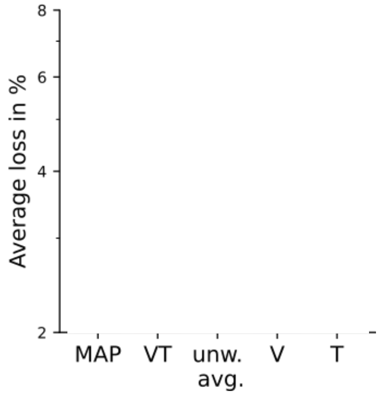




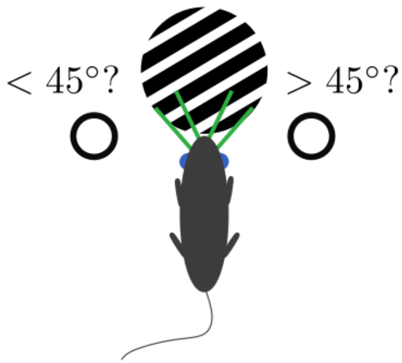
Learning Bayes-optimal inference of orientations from multimodal stimuli

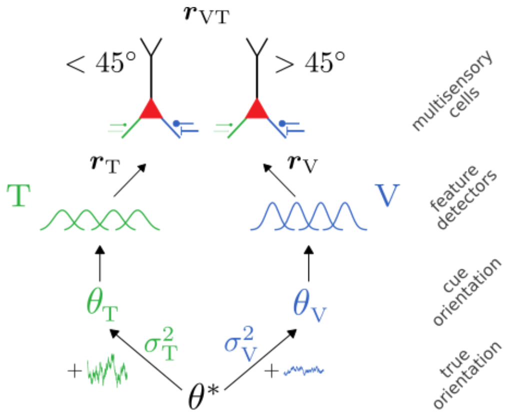


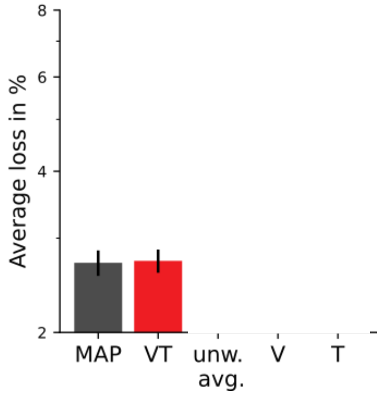




Learning Bayes-optimal inference of orientations from multimodal stimuli

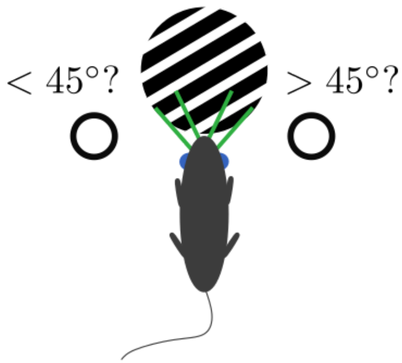


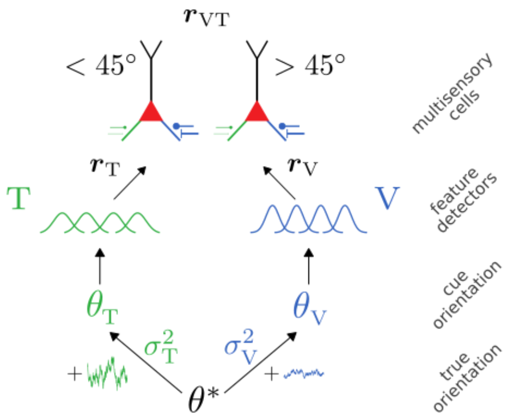


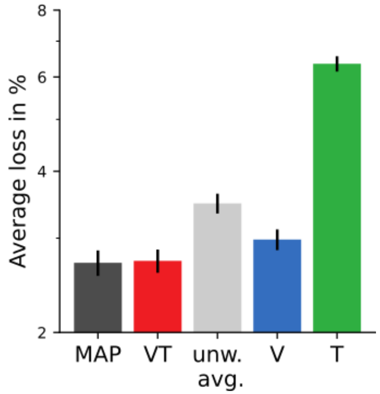


The trained model approximates ideal observers

Learning Bayes-optimal inference of orientations from multimodal stimuli

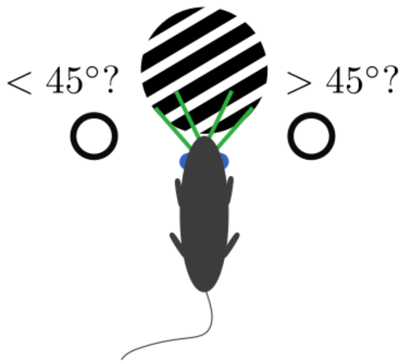


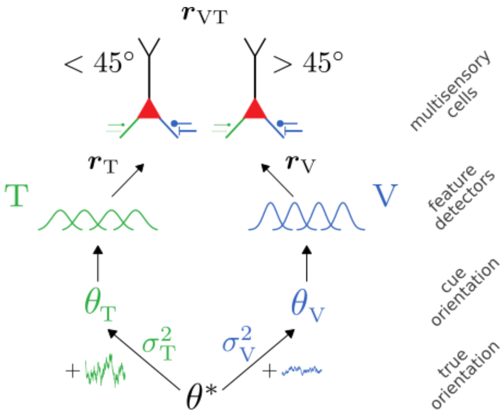


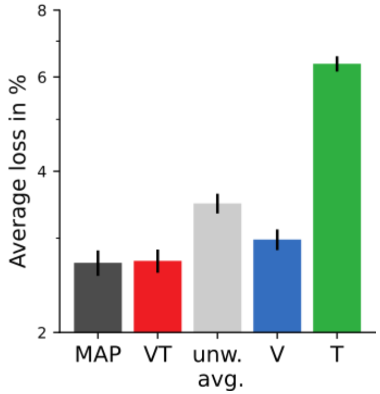


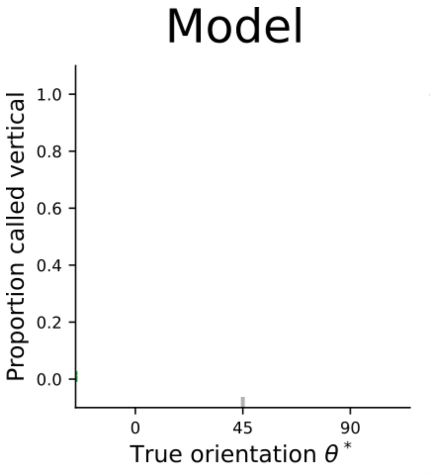
The trained model approximates ideal observers

Learning Bayes-optimal inference of orientations from multimodal stimuli



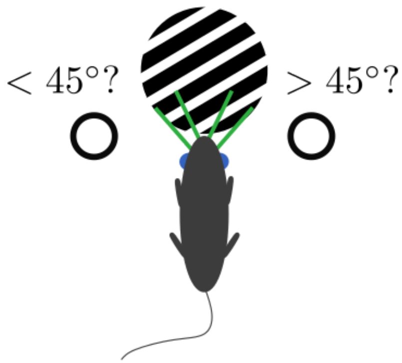


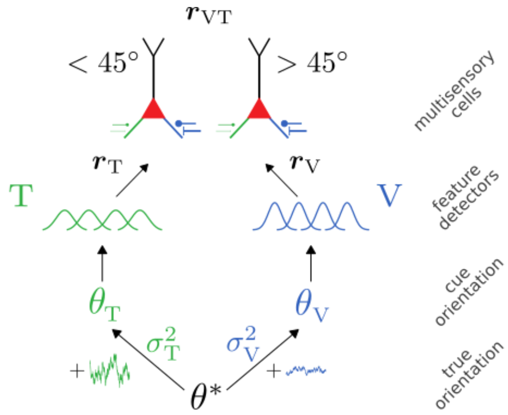


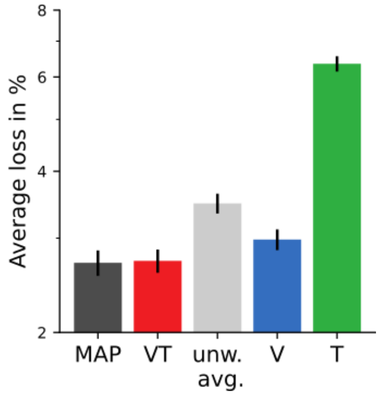


The trained model approximates ideal observers

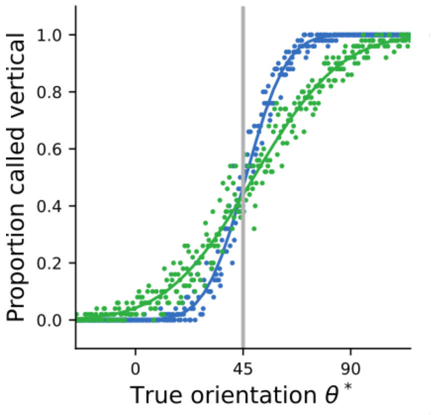
Learning Bayes-optimal inference of orientations from multimodal stimuli





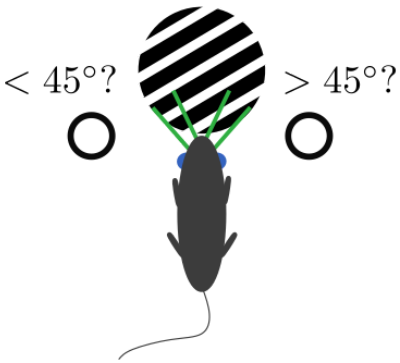


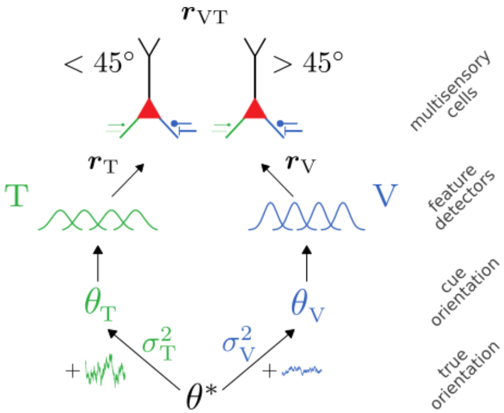
Model

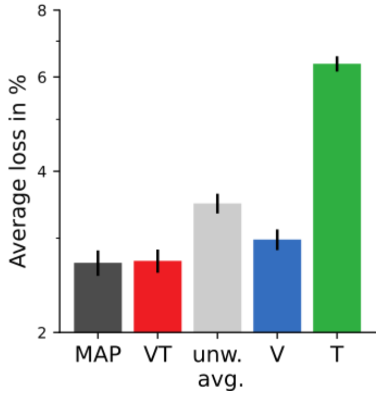


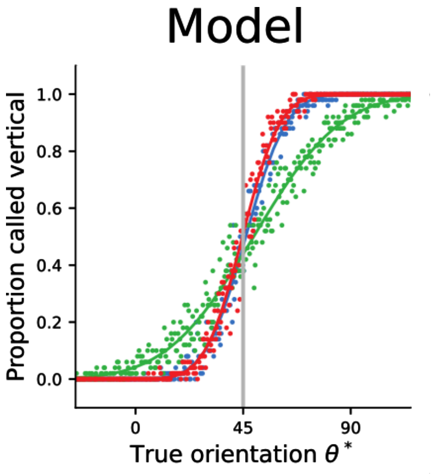
The trained model approximates ideal observers

Learning Bayes-optimal inference of orientations from multimodal stimuli



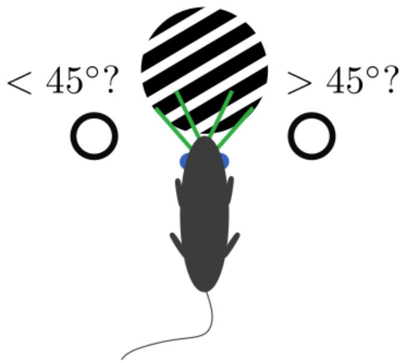


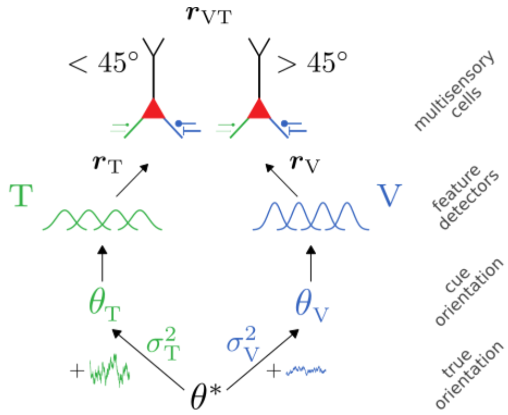


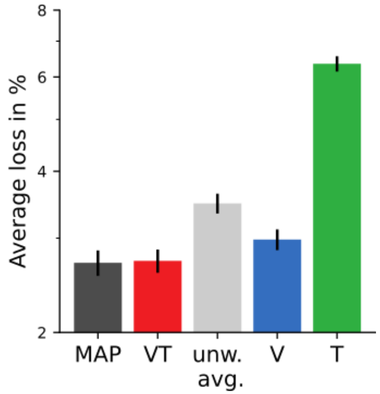


The trained model approximates ideal observers

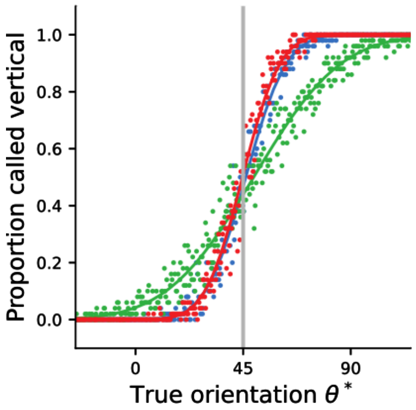
Learning Bayes-optimal inference of orientations from multimodal stimuli



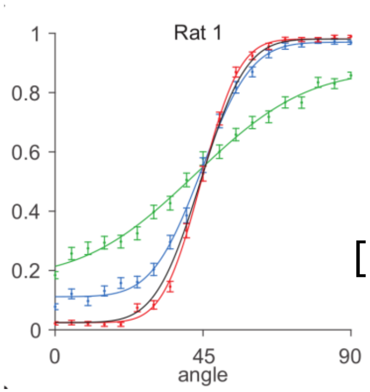




Model



Experiment

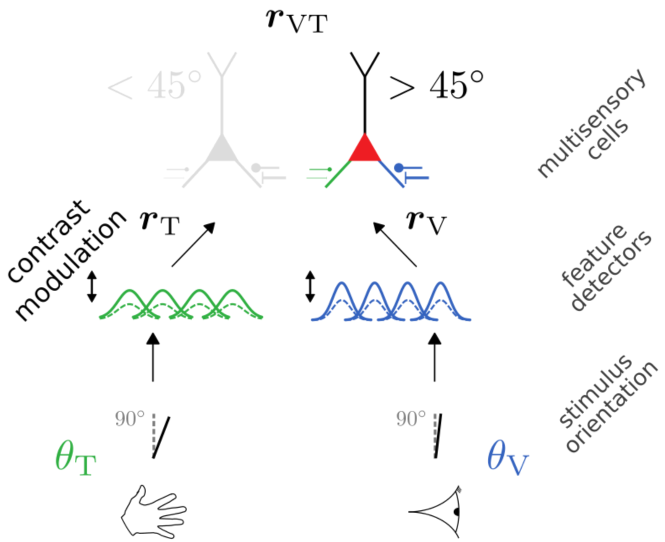


[Nikbakht et al., 2018]

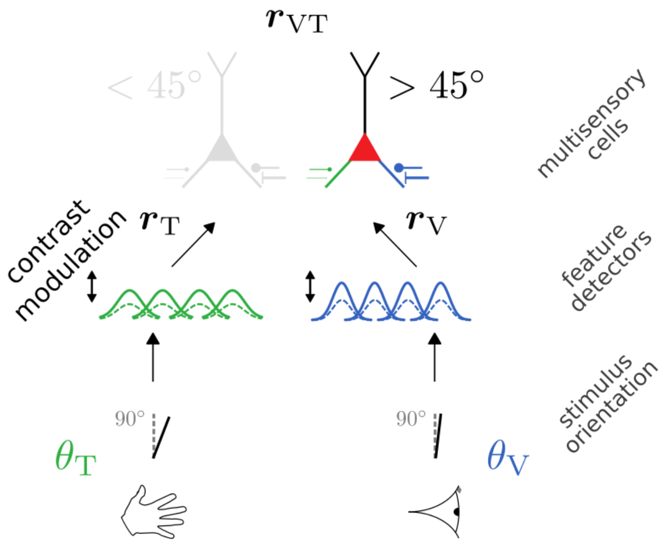
The trained model approximates ideal observers

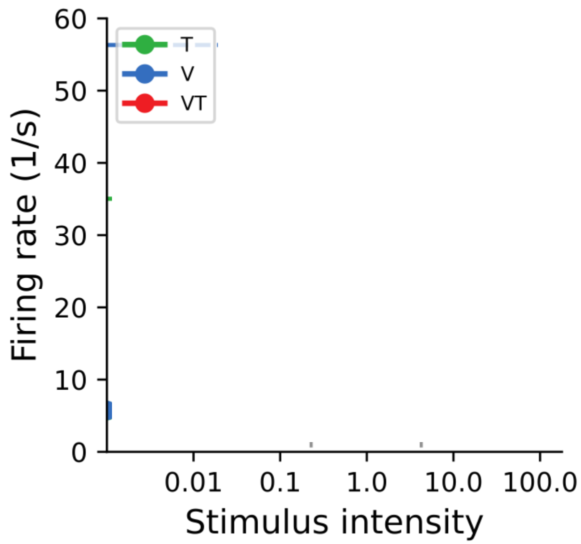
and reproduces psychophysical signatures of experimental data

Cross-modal suppression as reliability-weighted opinion pooling

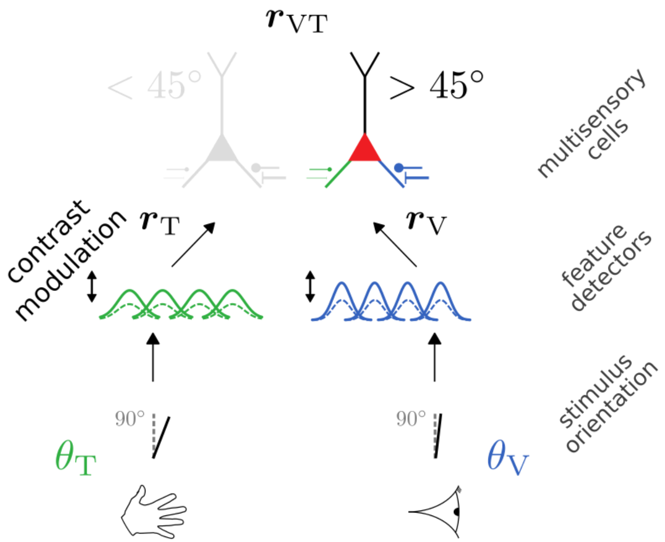


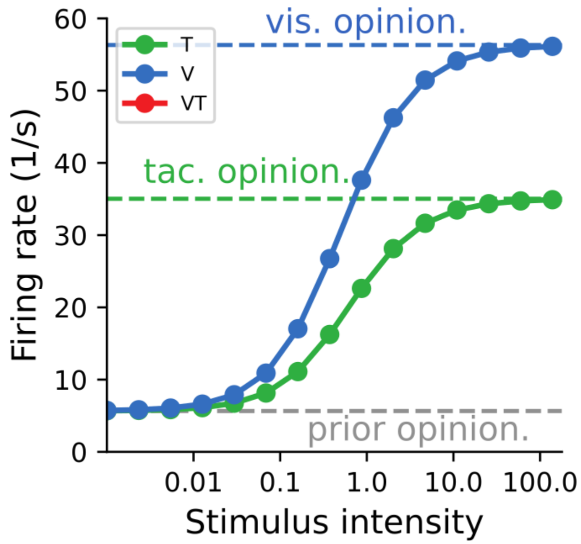
Cross-modal suppression as reliability-weighted opinion pooling



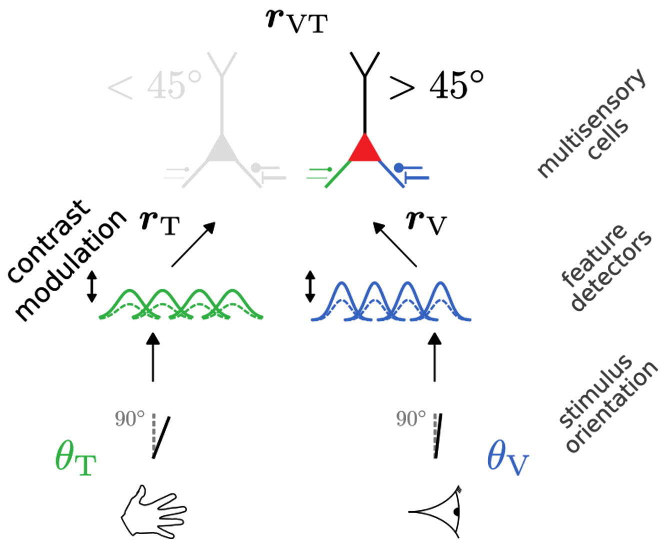


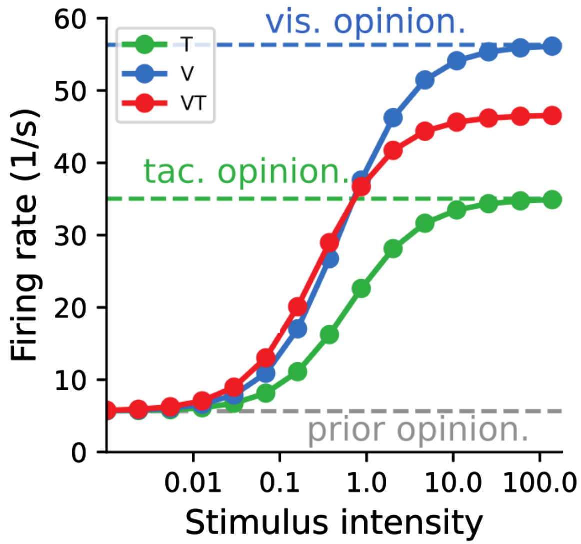
Cross-modal suppression as reliability-weighted opinion pooling



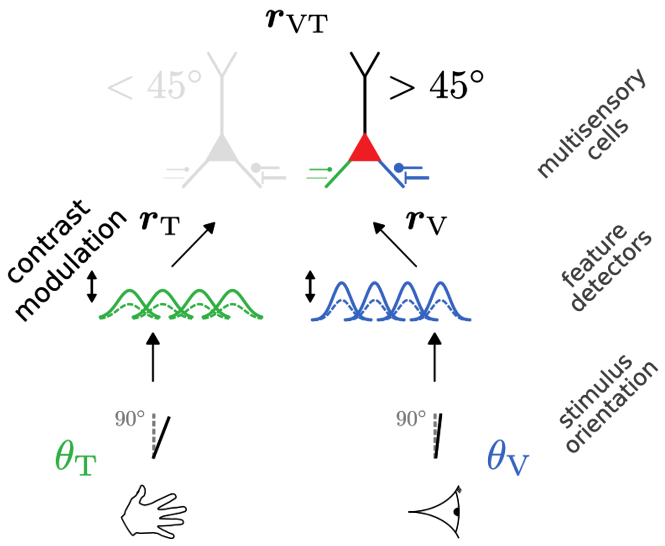


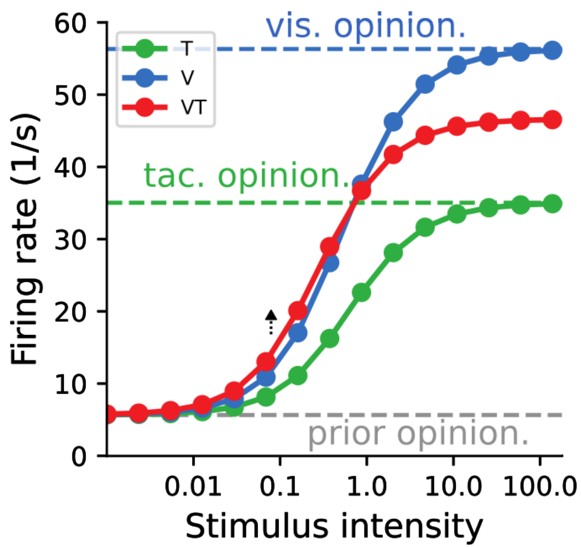
Cross-modal suppression as reliability-weighted opinion pooling





Cross-modal suppression as reliability-weighted opinion pooling

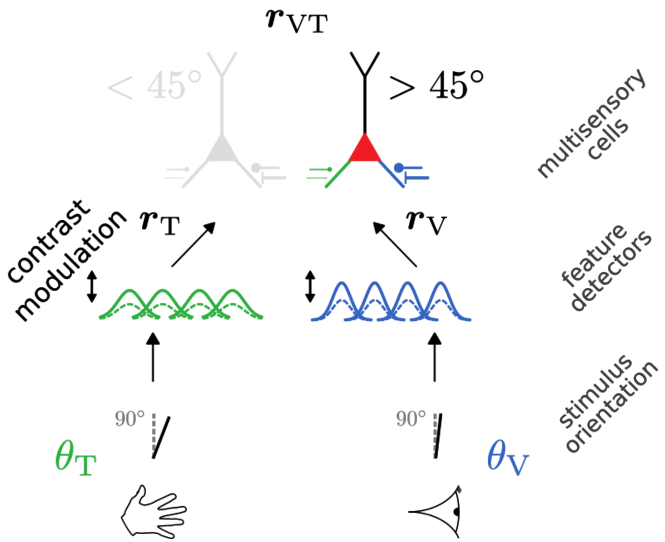


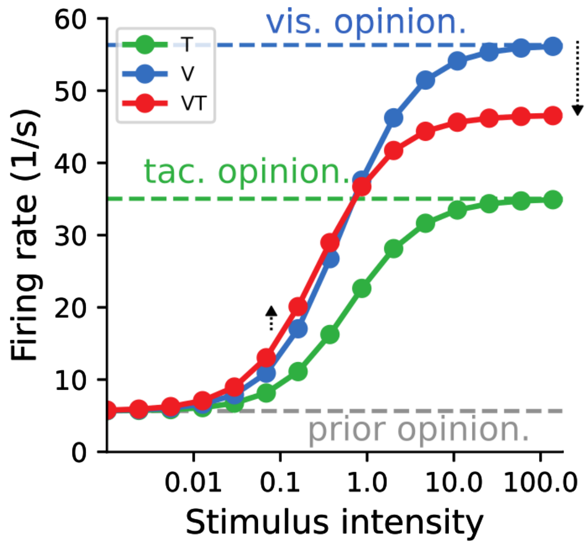


The trained model exhibits cross-modal suppression:

- at low stimulus intensities, firing rate is larger bimodal condition

Cross-modal suppression as reliability-weighted opinion pooling

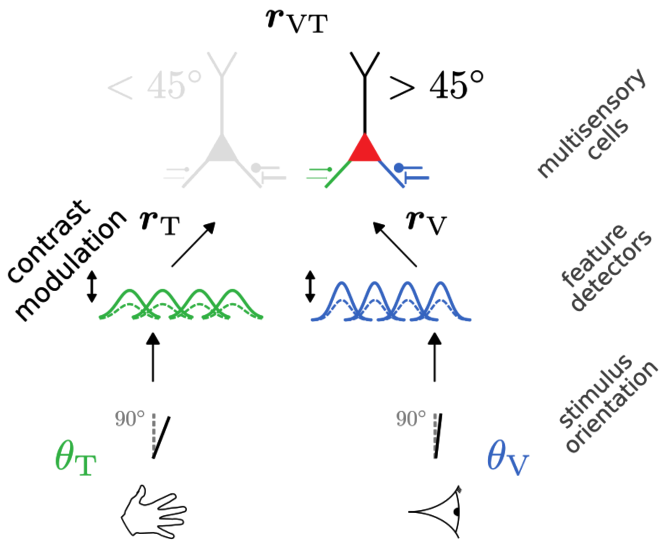


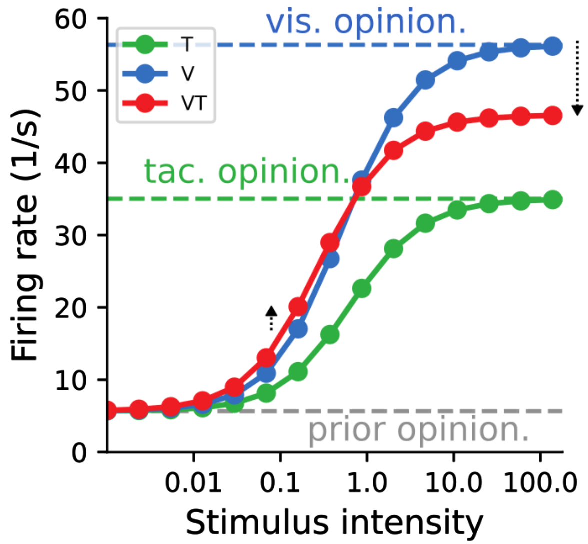


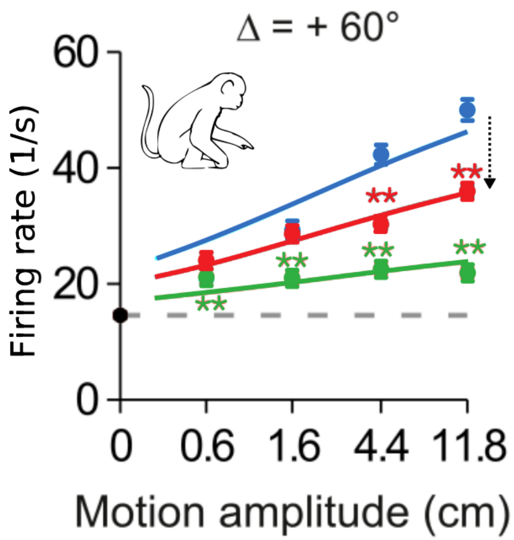
The trained model exhibits cross-modal suppression:

- at low stimulus intensities, firing rate is larger bimodal condition
- at high stimulus intensities, firing rate is smaller in bimodal condition

Cross-modal suppression as reliability-weighted opinion pooling





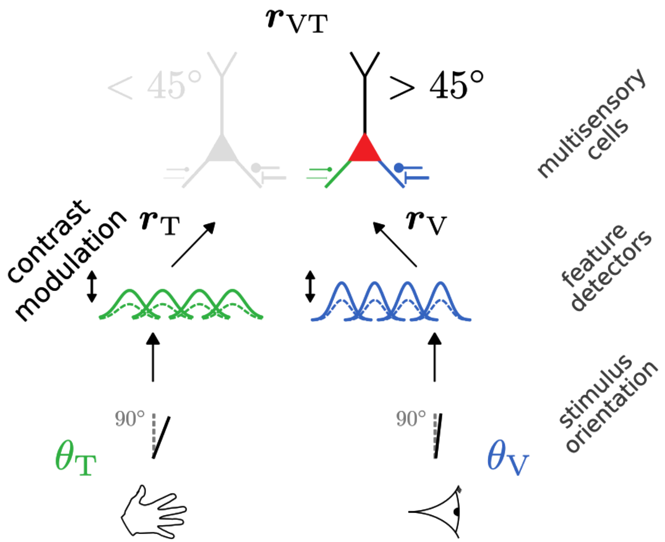


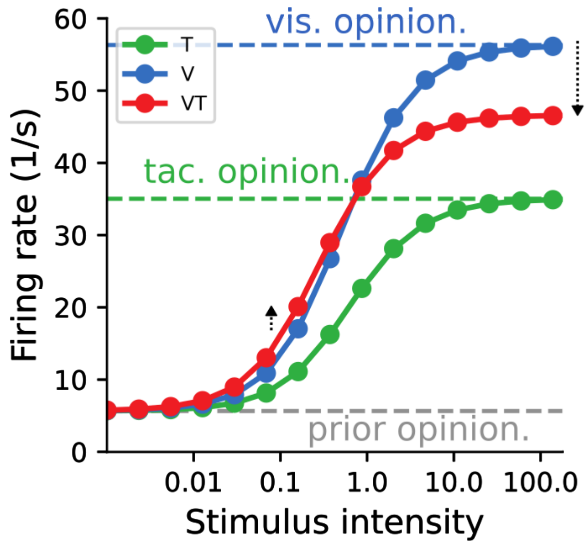
[Ohshiro et al., 2017]

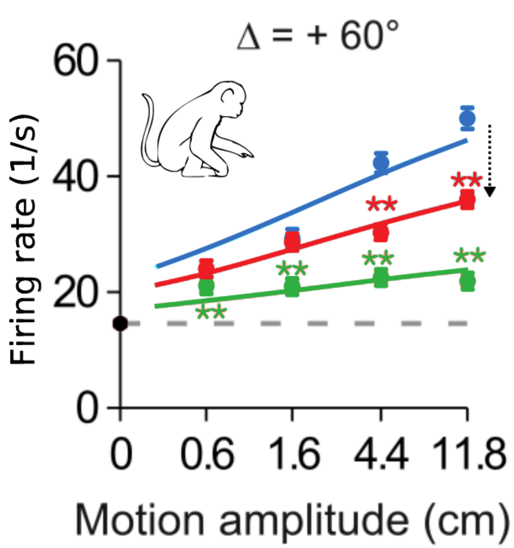
The trained model exhibits cross-modal suppression:

- at low stimulus intensities, firing rate is larger bimodal condition
- at high stimulus intensities, firing rate is smaller in bimodal condition

Cross-modal suppression as reliability-weighted opinion pooling







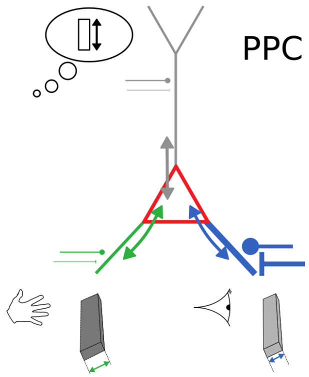
[Ohshiro et al., 2017]

The trained model exhibits cross-modal suppression:

- at low stimulus intensities, firing rate is larger bimodal condition
- at high stimulus intensities, firing rate is smaller in bimodal condition
- example prediction for experiments: strength of suppression depends on relative reliabilities of the two modalities

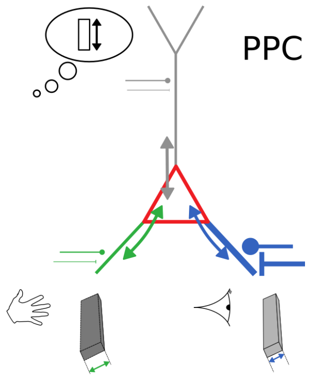
Summary & Outlook

- Neuron models with conductance-based synapses naturally implement computations required for probabilistic cue integration



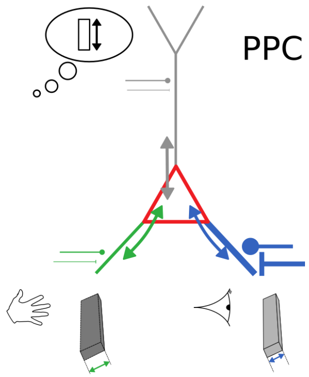
Summary & Outlook

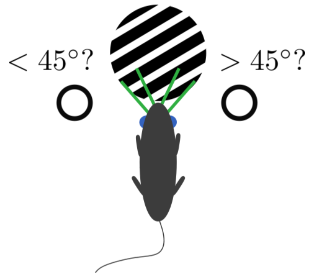
- Neuron models with conductance-based synapses naturally implement computations required for probabilistic cue integration
- Our plasticity rules matches the somatic potential distribution to a target distribution & weights pathways according to reliability



Summary & Outlook

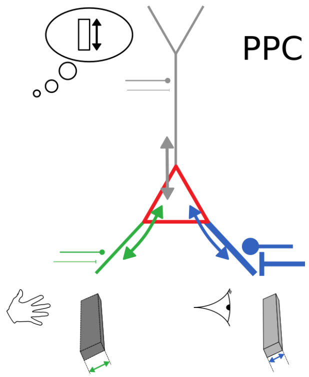
- Neuron models with conductance-based synapses naturally implement computations required for probabilistic cue integration
- Our plasticity rules matches the somatic potential distribution to a target distribution & weights pathways according to reliability
- A model trained in a multisensory cue integration tasks reproduces behavioral and neuronal experimental data

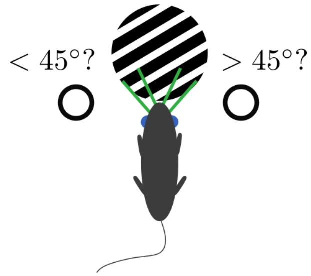


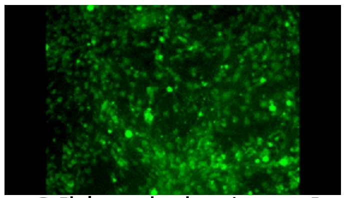


Summary & Outlook

- Neuron models with conductance-based synapses naturally implement computations required for probabilistic cue integration
- Our plasticity rules matches the somatic potential distribution to a target distribution & weights pathways according to reliability
- A model trained in a multisensory cue integration tasks reproduces behavioral and neuronal experimental data
- The direct connection between normative and mechanistic descriptions allows for predictions on the systems as well as cellular level
- Next: work out (new) detailed pre-/"post"dictions for specific experimental setups

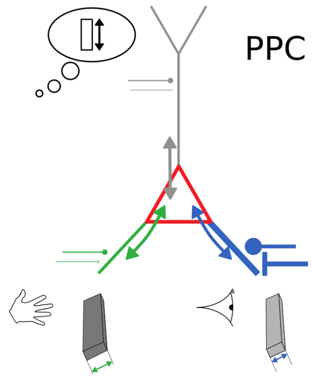


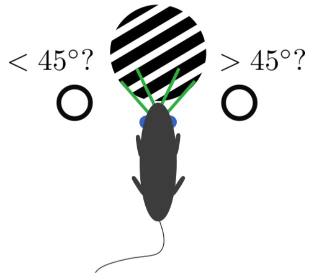


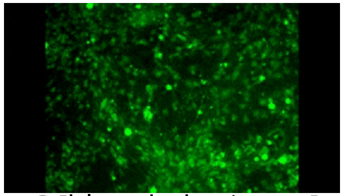

[blog-thebrain.org]

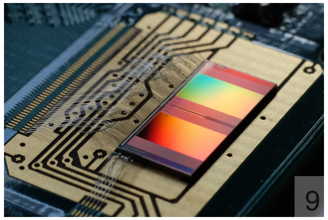
Summary & Outlook

- Neuron models with conductance-based synapses naturally implement computations required for probabilistic cue integration
- Our plasticity rules matches the somatic potential distribution to a target distribution & weights pathways according to reliability
- A model trained in a multisensory cue integration tasks reproduces behavioral and neuronal experimental data
- The direct connection between normative and mechanistic descriptions allows for predictions on the systems as well as cellular level
- Next: work out (new) detailed pre-/"post"dictions for specific experimental setups
- Analog neuromorphic systems present a fitting substrate: non-linear differential eq. tricky to integrate






[blog-thebrain.org]



[Billaudelle et al., 2020]