Conductance-based dendrites perform reliability-weighted opinion pooling

Jakob Jordan,
João Sacramento, Mihai A. Petrovici & Walter Senn

Department of Physiology, University of Bern, Switzerland
Kirchhoff-Institute for Physics, Heidelberg University, Germany
Institute of Neuroinformatics, UZH / ETH Zurich, Switzerland
Cue integration is a fundamental computational principle of cortex
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Neurons with *conductance-based synapses* naturally implement probabilistic cue integration.
An observation

Bayes-optimal inference

Estimated angle $\hat{\theta}$
An observation

Bayes-optimal inference
Bayes-optimal inference

mean $\mu$

precision $1/\sigma^2$

$$\mu = \frac{\mu_0}{\sigma_0^2} + \frac{\mu_1}{\sigma_1^2} + \frac{\mu_2}{\sigma_2^2}$$

$$\frac{1}{\sigma^2} = \frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$$
An observation

Bayes-optimal inference

mean $\mu$

precision $1/\sigma^2$

\[
\mu = \frac{\frac{\mu_0}{\sigma_0}}{\frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}} + \frac{\frac{\mu_1}{\sigma_1}}{\frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}} + \frac{\frac{\mu_2}{\sigma_2}}{\frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}}
\]

Bidirectional voltage dynamics

Estimated angle $\hat{\theta}$

top-down

bottom-up

$p_0$

$p_1$

$p_2$

$p$
An observation

Bayes-optimal inference

mean $\mu$

precision $1/\sigma^2$

$$\mu = \frac{\mu_0}{\sigma_0^2} + \frac{\mu_1}{\sigma_1^2} + \frac{\mu_2}{\sigma_2^2}$$

$$\frac{1}{\sigma^2} = \frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$$

Bidirectional voltage dynamics

membrane pot. $\overline{E}_s$

membrane cond. $\overline{g}_s$

$$\overline{E}_s = \frac{g_0 E_0 + g_1 E_1 + g_2 E_2}{g_0 + g_1 + g_2}$$

$$\overline{g}_s = g_0 + g_1 + g_2$$

$g_0 = g_0^I$

$g_1 = g_1^E + g_2^I$

$g_2 = g_2^E + g_2^I$
An observation

Bayes-optimal inference

\[ \mu = \frac{\frac{\mu_0}{\sigma_0^2} + \frac{\mu_1}{\sigma_1^2} + \frac{\mu_2}{\sigma_2^2}}{\frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}} \]

\[ \frac{1}{\sigma^2} = \frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} \]

Bidirectional voltage dynamics

membrane pot. $\vec{E}_s$
membrane cond. $\bar{g}_s$

\[ \vec{E}_s = \frac{g_0 E_0 + g_1 E_1 + g_2 E_2}{g_0 + g_1 + g_2} \]

\[ \bar{g}_s = g_0 + g_1 + g_2 \]

$g_0 = g_0^L$
$g_1 = g_1^E + g_2^I$
$g_2 = g_2^E + g_2^I$

opinion
\[ \mu \leftrightarrow \vec{E}_s \]
reliability
\[ \frac{1}{\sigma^2} \leftrightarrow \bar{g}_s \]
An observation

Bayes-optimal inference

mean $\mu$

precision $1/\sigma^2$

$$\mu = \frac{\mu_0}{\sigma_0^2} + \frac{\mu_1}{\sigma_1^2} + \frac{\mu_2}{\sigma_2^2}$$

$$\frac{1}{\sigma^2} = \frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$$

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$$\bar{g}_s = g_0 + g_1 + g_2$$

$g_0 = g_0^I$

$g_1 = g_1^E + g_2^I$

$g_2 = g_2^E + g_2^I$

---

opinion

$\mu \leftrightarrow \bar{E}_s$

reliability

$1/\sigma^2 \leftrightarrow \bar{g}_s$
An observation

Bayes-optimal inference

mean $\mu$

precision $1/\sigma^2$

$$\mu = \frac{\mu_0}{\sigma_0^2} + \frac{\mu_1}{\sigma_1^2} + \frac{\mu_2}{\sigma_2^2}$$

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Bidirectional voltage dynamics

membrane pot. $\bar{E}_s$

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$$\bar{E}_s = \frac{g_0 E_0 + g_1 E_1 + g_2 E_2}{g_0 + g_1 + g_2}$$

$$\bar{g}_s = g_0 + g_1 + g_2$$

$g_0 = g_0^l$

$g_1 = g_1^E + g_2^I$

$g_2 = g_2^E + g_2^I$
Membrane potential dynamics as noisy gradient ascent

\[ p \]
\[ u_s \]

\[ p_0 \]
\[ u_s \]

\[ p_1 \]
\[ u_s \]

\[ p_2 \]
\[ u_s \]

opinion
\[ \mu \leftrightarrow \bar{E}_s \]
reliability
\[ \frac{1}{\sigma^2} \leftrightarrow \bar{g}_s \]
Membrane potential dynamics as noisy gradient ascent

\[ p \propto p_0 p_1 p_2 \ldots \]
Membrane potential dynamics as noisy gradient ascent

\[ p \propto p_0 p_1 p_2 \ldots \]

\[ p(u_s|W, r) = \frac{1}{Z'} \prod_{d=0}^{D} p_d(u_s|W_d, r) \]
\[ = \frac{1}{Z} e^{-\frac{\tilde{g}_s}{2}(u_s-\bar{E}_s)^2} \]
Membrane potential dynamics as noisy gradient ascent

\[ p(u_s | W, r) = \frac{1}{Z'} \prod_{d=0}^{D} p_d(u_s | W_d, r) = \frac{1}{Z} e^{-\frac{g_s}{2}} (u_s - \bar{E}_s)^2 \]

\[ C u_s = \frac{\partial}{\partial u_s} \log p(u_s | W, r) + \xi \]
\[ = \sum_{d=0}^{D} \left( g^L_d (E^L - u_s) + g^E_d (E^E - u_s) + g^I_d (E^I - u_s) \right) + \xi \]
Membrane potential dynamics as noisy gradient ascent

\[
p \propto p_0 p_1 p_2 \ldots
\]

\[p(u_s|W, r) = \frac{1}{Z'} \prod_{d=0}^{D} p_d(u_s|W_d, r) = \frac{1}{Z} e^{-\frac{\bar{\sigma}_s}{2}(u_s-\bar{E}_s)^2}
\]

\[
C \dot{u}_s = \frac{\partial}{\partial u_s} \log p(u_s|W, r) + \xi
= \sum_{d=0}^{D} \left( g_d^L (E^L - u_s) + g_d^E (E^E - u_s) + g_d^I (E^I - u_s) \right) + \xi
\]

\[
\mathbb{E}[u_s] = \bar{E}_s
\]

Average membrane potentials
== reliability-weighted opinions
Membrane potential dynamics as noisy gradient ascent

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\[ C u_s = \frac{\partial}{\partial u_s} \log p(u_s | W, r) + \xi \]

\[ = \sum_{d=0}^D (g_d^L (E^L - u_s) + g_d^E (E^E - u_s) + g_d^I (E^I - u_s)) + \xi \]

Average membrane potentials

\[ \bar{E}[u_s] = \bar{E}_s \]

== reliability-weighted opinions

Membrane potential variance

\[ \text{Var}[u_s] = \frac{1}{\bar{g}_s} \]

== 1/total reliability
Stochastic-gradient-ascent-based synaptic plasticity

\[ p(u_s | W, r) \]

\[ p^*(u_s) \]
Stochastic-gradient-ascent-based synaptic plasticity

\[ \dot{W}_{d}^{E/I} \propto \frac{\partial}{\partial W_{d}^{E/I}} \log p(u_{s}^{*}|W, r) \]

\[ u_{s}^{*}: \text{sample from target distribution } p^{*}(u_{s}) \]
Stochastic-gradient-ascent-based synaptic plasticity

\[ \dot{W}_{d}^{E/I} \propto \frac{\partial}{\partial W_{d}^{E/I}} \log p(u^*_s|W, r) \]

\[ \propto [ \Delta \mu^{E/I} + \Delta \sigma^2 ] r \]

\( u^*_s \): sample from target distribution \( p^*(u_s) \)
Stochastic-gradient-ascent-based synaptic plasticity

\[ \dot{W}_d^{E/I} \propto \frac{\partial}{\partial W_d^{E/I}} \log p(u_s^*|W, r) \]

\[ \propto \left[ \Delta \mu^{E/I} + \Delta \sigma^2 \right] r \]

\[ \Delta \mu^{E/I} \propto (u_s^* - \bar{E}_s) (E^{E/I} - \bar{E}_s) \]

\[ u_s^*: \text{sample from target distribution } p^*(u_s) \]
Stochastic-gradient-ascent-based synaptic plasticity

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\dot{W}_{d}^{E/I} \propto \frac{\partial}{\partial W_{d}^{E/I}} \log p(u_{s}^{*}|W, r) \\
\propto [\Delta \mu^{E/I} + \Delta \sigma^2] r
\]

\[
\Delta \mu^{E/I} \propto (u_{s}^{*} - \bar{E}_{s}) (E^{E/I} - \bar{E}_{s})
\]

\[
\Delta \sigma^2 \propto \frac{1}{2} \left( \frac{1}{\bar{g}_{s}} - (u_{s}^{*} - \bar{E}_{s})^2 \right)
\]

\(u_{s}^{*}\): sample from target distribution \(p^{*}(u_{s})\)
Stochastic-gradient-ascent-based synaptic plasticity

\[ \dot{W}_{d}^{E/I} \propto \frac{\partial}{\partial W_{d}^{E/I}} \log p(u_{s}^{*}|W, r) \]

\[ \propto [ \Delta \mu_{E/I} + \Delta \sigma^2 ] r \]

\[ \Delta \mu_{E/I} \propto (u_{s}^{*} - \bar{E}_{S}) (E_{E/I}^{E} - \bar{E}_{S}) \]

\[ \Delta \sigma^2 \propto \frac{1}{2} \left( \frac{1}{\bar{g}_{s}} - (u_{s}^{*} - \bar{E}_{S})^2 \right) \]

Synaptic plasticity modifies excitatory/inhibitory synapses

- in approx. opposite directions to match the mean
- in identical directions to match the variance

\( u_{s}^{*} \): sample from target distribution \( p^{*}(u_{s}) \)
Synaptic plasticity performs error-correction and reliability matching.
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Early learning:
- $\mu(u) = E_u \rightarrow \mu(u^*)$
- $\sigma^2(u) = \frac{1}{\theta_u} \rightarrow \sigma^2(u^*)$

Late learning:
- Error-corrected
- Reliability-matched

Graphs showing membrane potential, $W_{E}^E + W_{d}$, and $r$ over time.
Synaptic plasticity performs error-correction and reliability matching.
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Synaptic plasticity performs error-correction and reliability matching.

Early learning:
- Dendritic predictive plasticity
- \( \mu(u) = \tilde{E}_u \rightarrow \mu(u') \)
- \( \sigma^2(u) = \frac{1}{\beta_u} \rightarrow \sigma^2(u') \)

Late learning:
- Error-corrected
- Reliability-matched
Learning Bayes-optimal inference of orientations from multimodal stimuli
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The trained model approximates ideal observers and reproduces psychophysical signatures of experimental data.
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Model

The trained model approximates ideal observers.
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[Nikbakht et al., 2018]
Cross-modal suppression as reliability-weighted opinion pooling
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The trained model exhibits cross-modal suppression:

- at low stimulus intensities, firing rate is larger bimodal condition
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- at high stimulus intensities, firing rate is smaller in bimodal condition
Cross-modal suppression as reliability-weighted opinion pooling

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- at low stimulus intensities, firing rate is larger in bimodal condition
- at high stimulus intensities, firing rate is smaller in bimodal condition

[Ohshiro et al., 2017]
Cross-modal suppression as reliability-weighted opinion pooling

The trained model exhibits cross-modal suppression:

- at low stimulus intensities, firing rate is larger in bimodal condition
- at high stimulus intensities, firing rate is smaller in bimodal condition
- example prediction for experiments: strength of suppression depends on relative reliabilities of the two modalities

[Ohshiro et al., 2017]
Summary & Outlook

- Neuron models with conductance-based synapses naturally implement computations required for probabilistic cue integration
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Summary & Outlook

- Neuron models with conductance-based synapses naturally implement computations required for probabilistic cue integration.
- Our plasticity rules matches the somatic potential distribution to a target distribution & weights pathways according to reliability.
- A model trained in a multisensory cue integration tasks reproduces behavioral and neuronal experimental data.
- The direct connection between normative and mechanistic descriptions allows for predictions on the systems as well as cellular level.
- Next: work out (new) detailed pre-/"post"dictions for specific experimental setups.
Summary & Outlook

- Neuron models with conductance-based synapses naturally implement computations required for probabilistic cue integration
- Our plasticity rules matches the somatic potential distribution to a target distribution & weights pathways according to reliability
- A model trained in a multisensory cue integration tasks reproduces behavioral and neuronal experimental data
- The direct connection between normative and mechanistic descriptions allows for predictions on the systems as well as cellular level
- Next: work out (new) detailed pre-/"post"dictions for specific experimental setups
- Analog neuromorphic systems present a fitting substrate: non-linear differential eq. tricky to integrate

[Billaudelle et al., 2020]