Optimal Oscillator Memory Networks

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intel

Motivation

Attractor networks are important models of memory in neuroscience & ML

- Memory networks models for hippocampus and other brain areas
- Error-correction in Vector Symbolic Architectures (VSA)
- “Modern Hopfield Networks” in Transformer networks

Many traditional attractor models are inefficient

- How to design efficient associative memories for neuromorphic hardware or coupled oscillators?

Classical "Hopfield" Associative Memory

An associative memory stores a set of patterns for robust recall

Binary Patterns

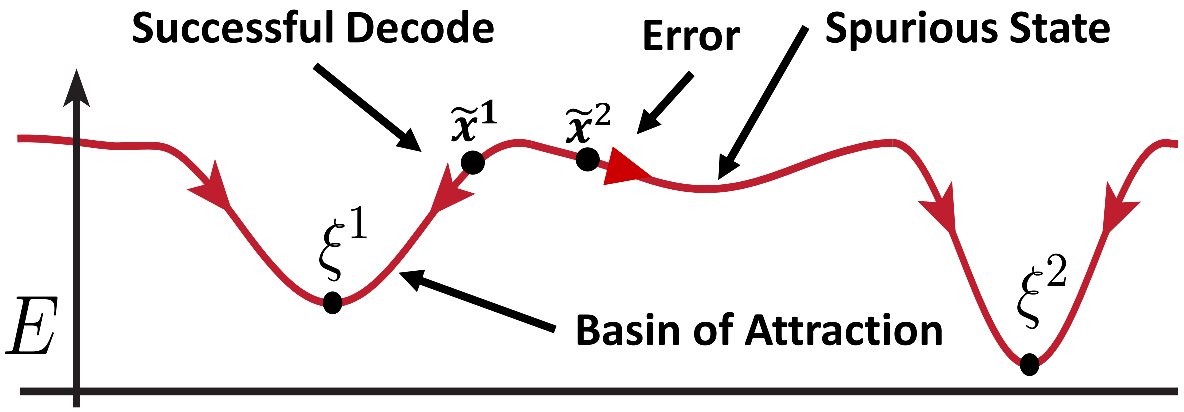
$\boldsymbol{\xi}^{\boldsymbol{k}} \in \{-1, 1\}^{N}$

Outer Product Learning Rule

$$\boldsymbol{W} = \frac{1}{M}\sum_{k=1}^{M} \boldsymbol{\xi}^{\boldsymbol{k}}\boldsymbol{\xi}^{\boldsymbol{k}T}$$

Energy Function

$E = -\boldsymbol{x}^{T}\boldsymbol{W}\boldsymbol{x}$



System Dynamics

$\Delta \boldsymbol{x} = f(-\nabla_{\boldsymbol{x}} E(\boldsymbol{x}))$

$\boldsymbol{x}(t+1) = f(\boldsymbol{W}\boldsymbol{x}(t))$

Error-Correction (Two Components)

- Collective State Computation
- State Quantization $f(x) = \text{sign}(x)$

Hopfield, John J. "Neural networks and physical systems with emergent collective computational abilities." *Proceedings of the national academy of sciences* 79.8 (1982): 2554-2558.

Efficiency of Associative Memories

Parameters

M	Patterns
N	Units/Neurons
$S = \lvert\{W_{ij}\}\rvert$	Synapses/Parameters
I_p	Pattern Information (Bits/Pattern)

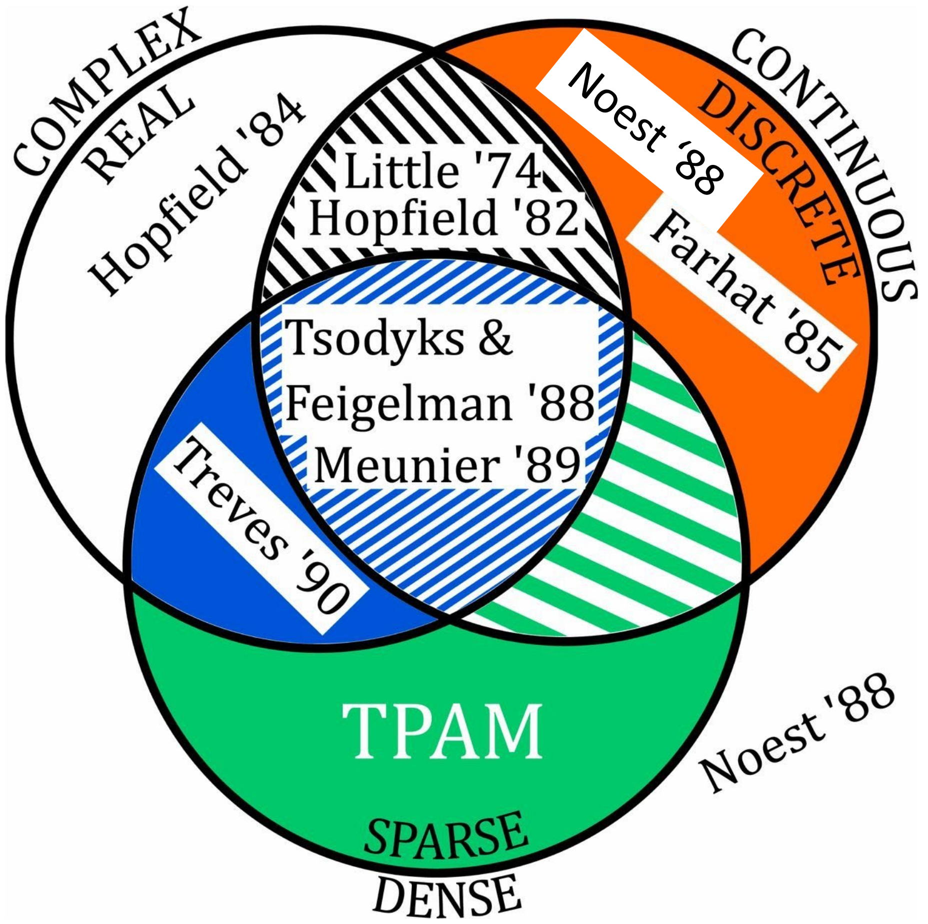
Pattern Capacity

$$\frac{M}{N} \quad \left[\frac{\text{Patterns}}{\text{Unit}}\right]$$

Information Capacity

$$\frac{I_p M}{S} \quad \left[\frac{\text{Bits}}{\text{Synapse}}\right]$$

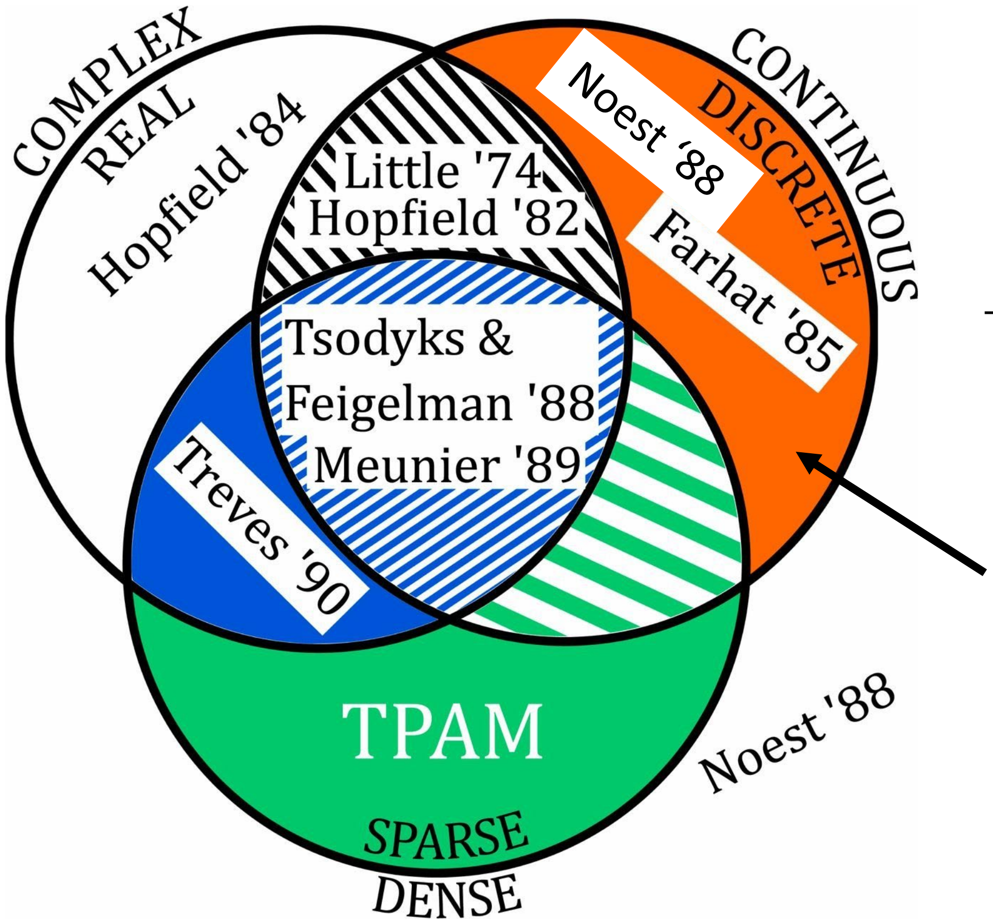
Existing Associative Memory Models



	Pattern Information	Pattern & Information Capacity
Binary & Dense & Discrete	High	Low
Sparse	Low	High
Complex & Dense & Continuous	High	Low

Frady, E. Paxon, and Friedrich T. Sommer. "Robust computation with rhythmic spike patterns." *Proceedings of the National Academy of Sciences* 116.36 (2019): 18050-18059.

Existing Associative Memory Models



	Pattern Complexity	Pattern & Information Capacity
Binary & Dense & Discrete	High	Low
Sparse	Low	High
Complex & Dense & Continuous	High	Low
Complex & Dense & Discrete	High	?

Frady, E. Paxon, and Friedrich T. Sommer. "Robust computation with rhythmic spike patterns." *Proceedings of the National Academy of Sciences* 116.36 (2019): 18050-18059.

Complex-Valued Phasor Associative Memory

Continuous Patterns
$\boldsymbol{\xi}^{\boldsymbol{k}} \in \mathbb{C}^N$

Complex Outer Product Learning Rule

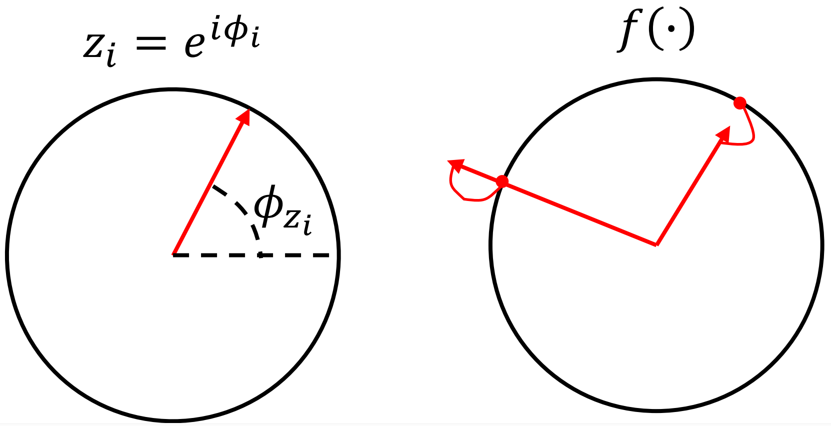
$$\boldsymbol{W} = \frac{1}{M}\sum_{k=1}^{M} \boldsymbol{\xi}^{\boldsymbol{k}}\left(\boldsymbol{\xi}^{k}\right)^{*\mathrm{T}}$$

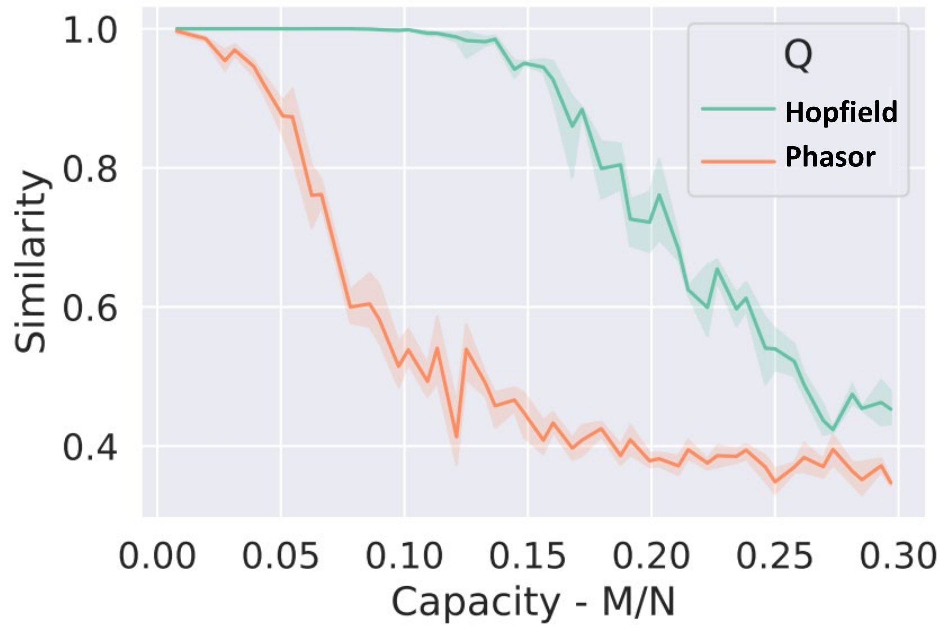
Energy
$\boldsymbol{E} = -(\boldsymbol{z})^{*\mathrm{T}}\boldsymbol{W}\boldsymbol{z}$

Dynamics

$$\boldsymbol{z}(t+1) = f\big(\boldsymbol{W}\boldsymbol{z}(t)\big)$$

$$f(z_i) = \frac{z_i}{|z_i|}$$





Noest, Andre. "Phasor neural networks." *Neural information processing systems*. 1987.

Q-State Phasor Neural Networks

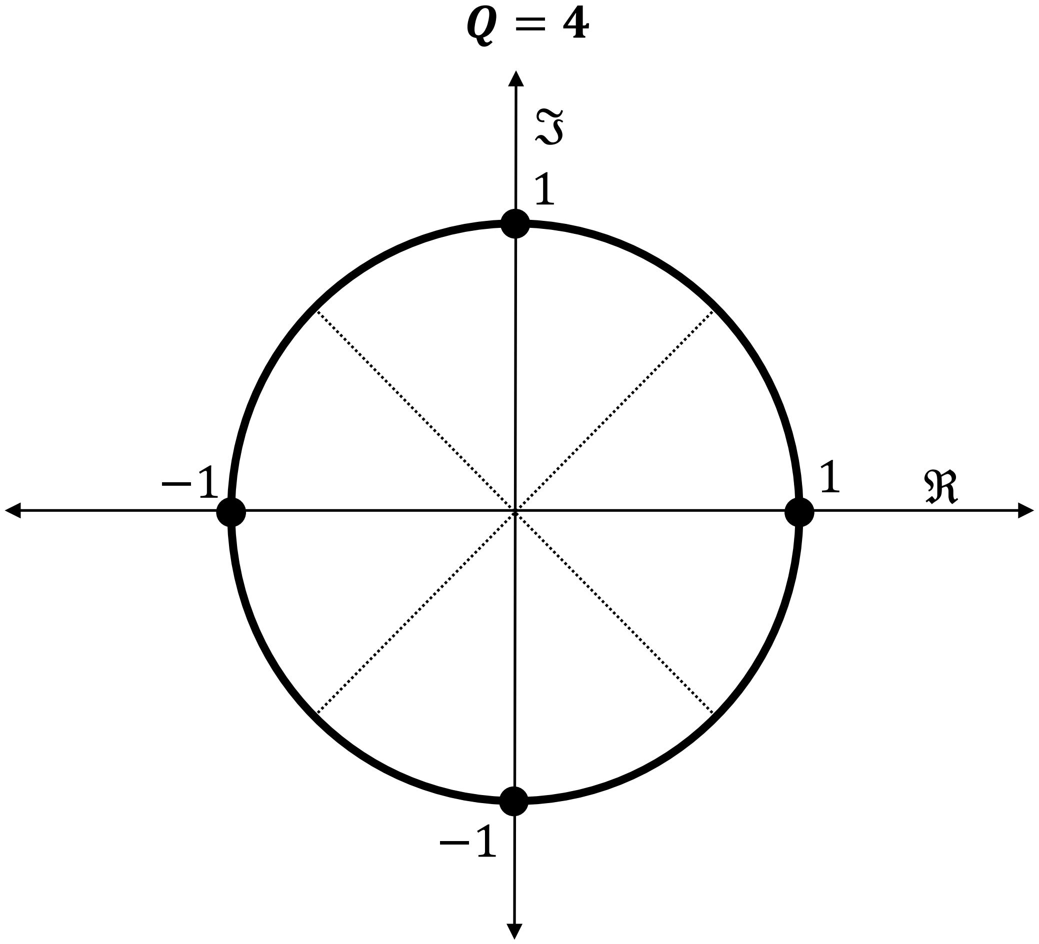
Patterns

$$\boldsymbol{\xi}^k = \exp\left(i\frac{2\pi \boldsymbol{q}^k}{Q}\right)$$

Dynamics

$$\boldsymbol{z}(t+1) = f\big(\boldsymbol{W}\boldsymbol{z}(t)\big)$$

$$f(u_i) = \exp\left(i\frac{2\pi}{Q}\operatorname{argmin}_q\left|\phi_i^{\mathrm{u}} - \frac{2\pi q}{Q}\right|\right)$$

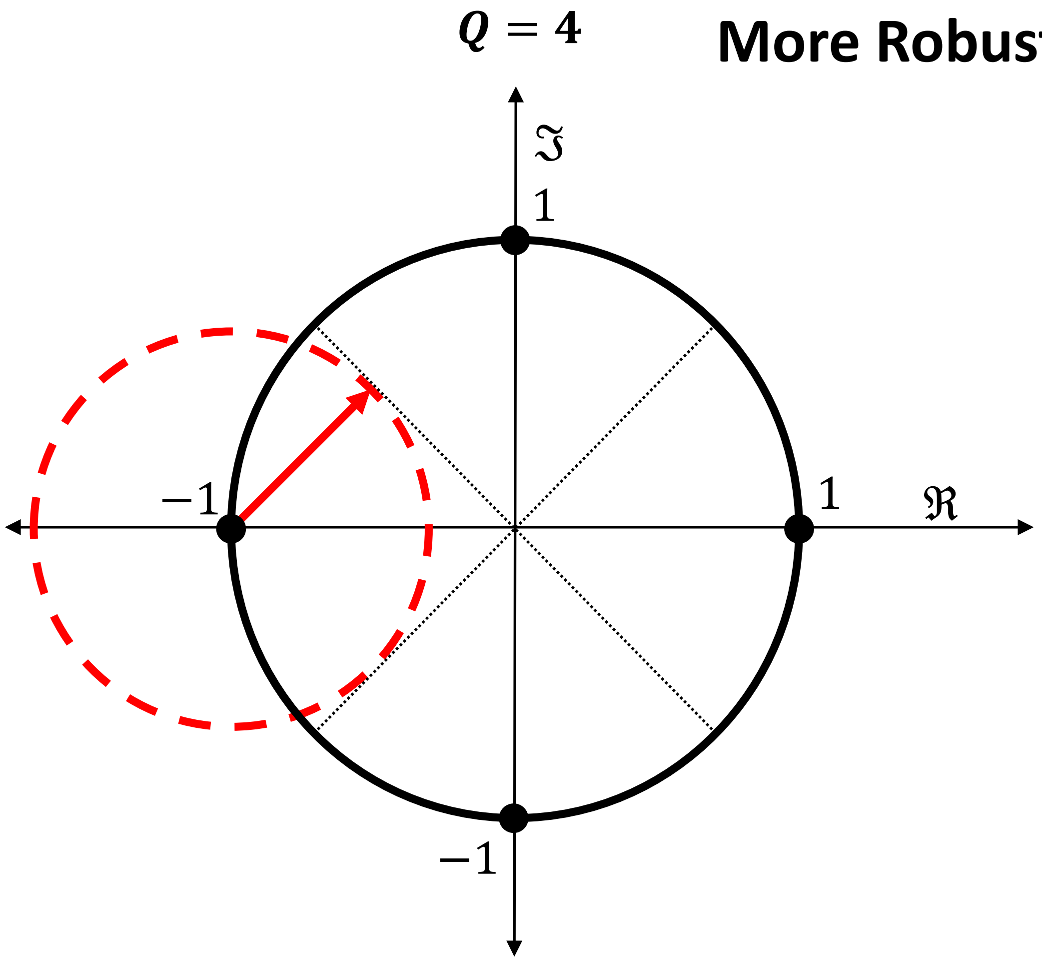


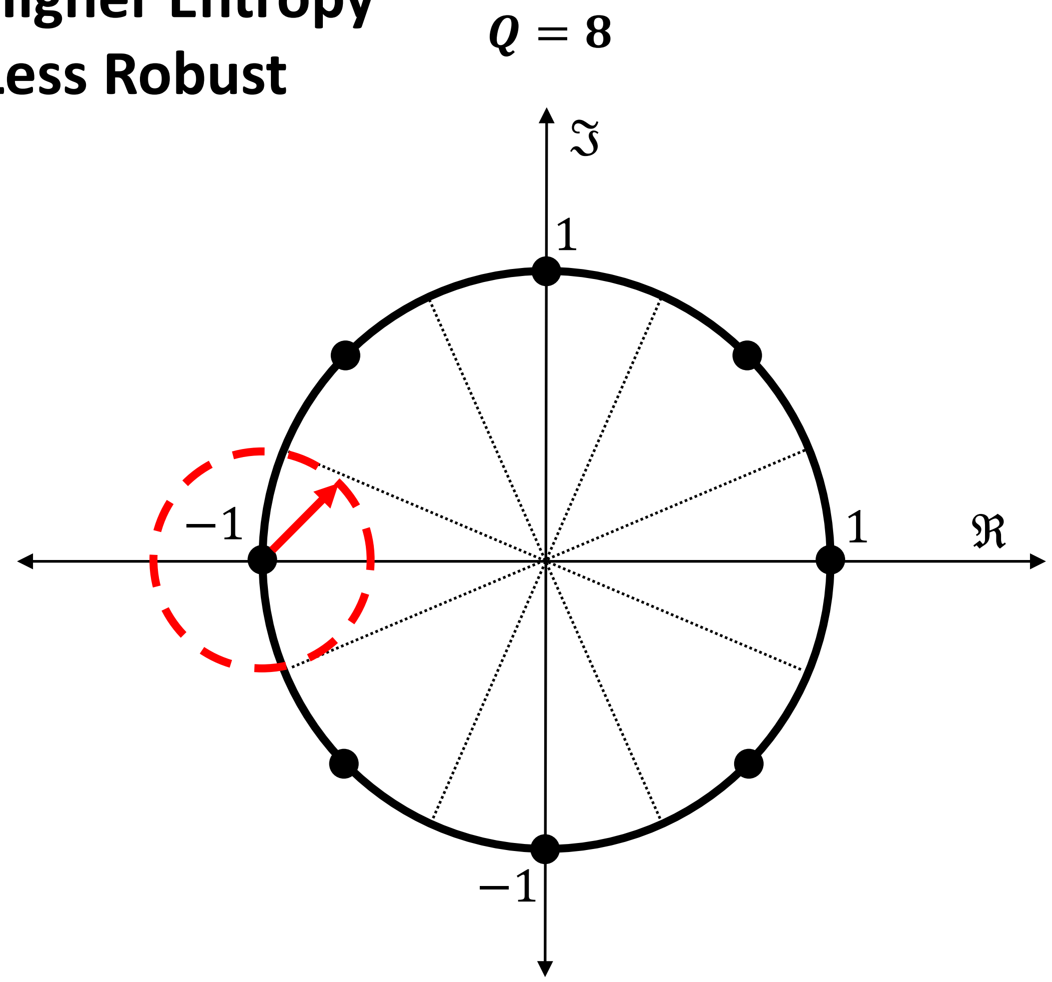
Noest, A. J. (1988). Discrete-state phasor neural networks. *Physical Review A*, *38*(4), 2196.

Trade-off Pattern Complexity vs. Error-Correction

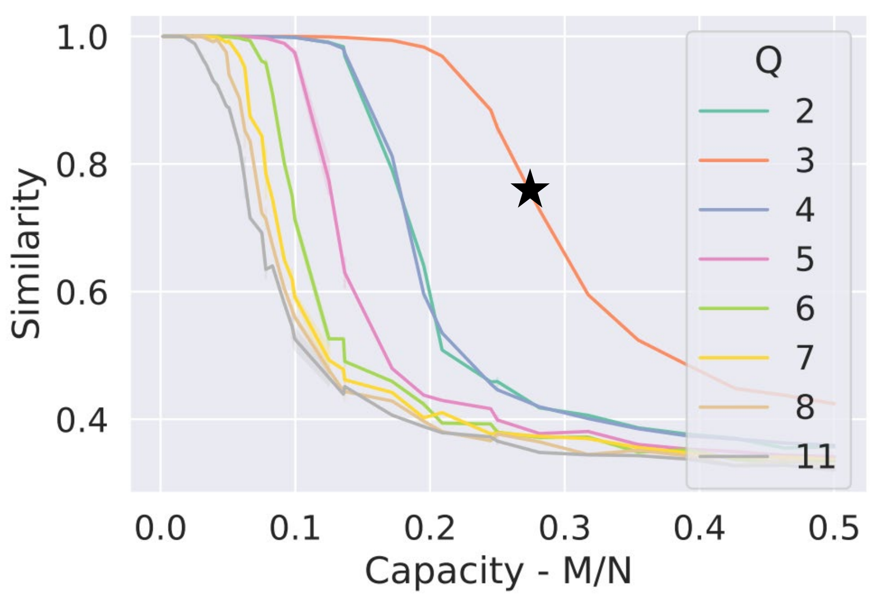
Lower Entropy
More Robust

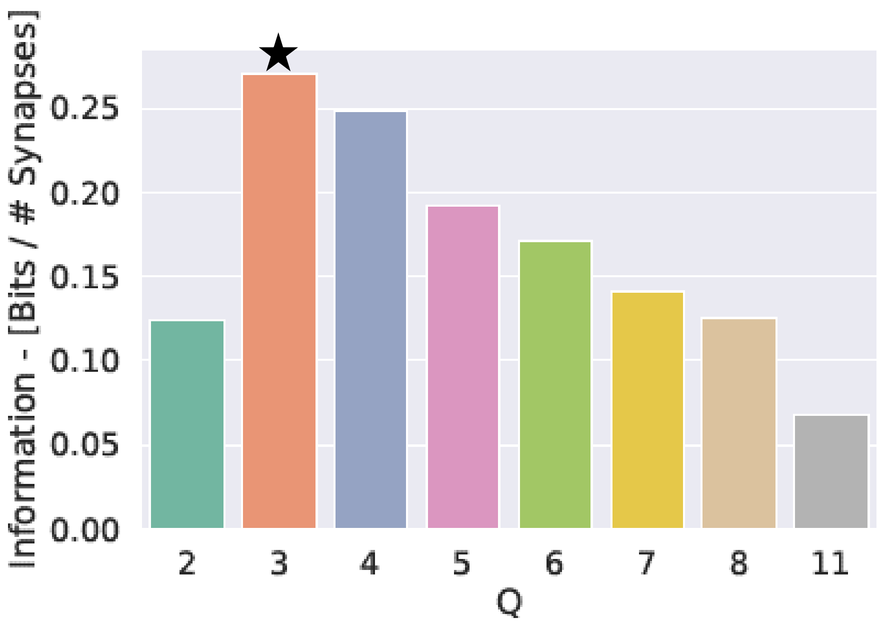
Higher Entropy
Less Robust





Capacity Results for Q-State Phasor Networks





★ Maximum capacity at Q=3 as predicted by mean-field theory - Cook, J. (1989)

Oscillator Networks

Mapping Associative Memories to Hardware

Synchronization in Weakly-Coupled Oscillators

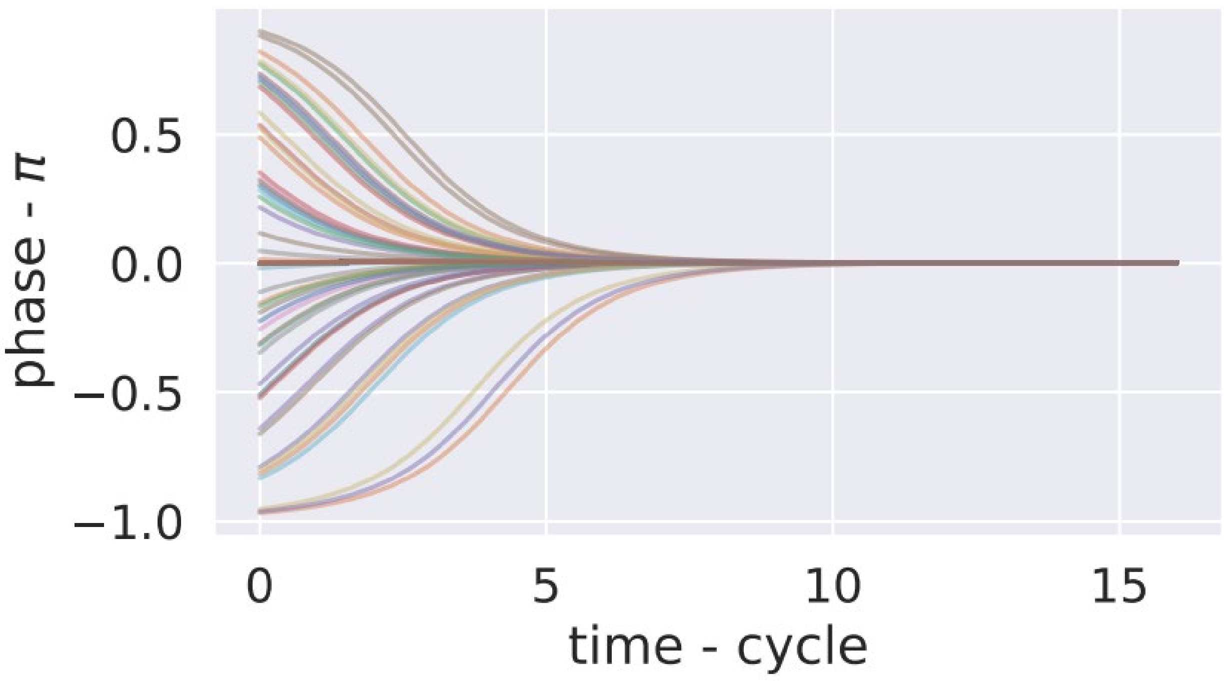
Continuous Phasors

$\boldsymbol{z}(t+1) = f(\boldsymbol{W}\boldsymbol{z}(t)); \; W_{ij} = 1$

$f(z_i) = \frac{z_i}{|z_i|}$

Kuramoto Model

$\dot{\phi}_i = \epsilon \sum_j \sin(\phi_j - \phi_i)$



Kuramoto Phasor Associative Memory

Phasor associative memories map to Kuramoto oscillator networks

Without state quantization fixed-points of dynamical system **ARE NOT*** stored patterns

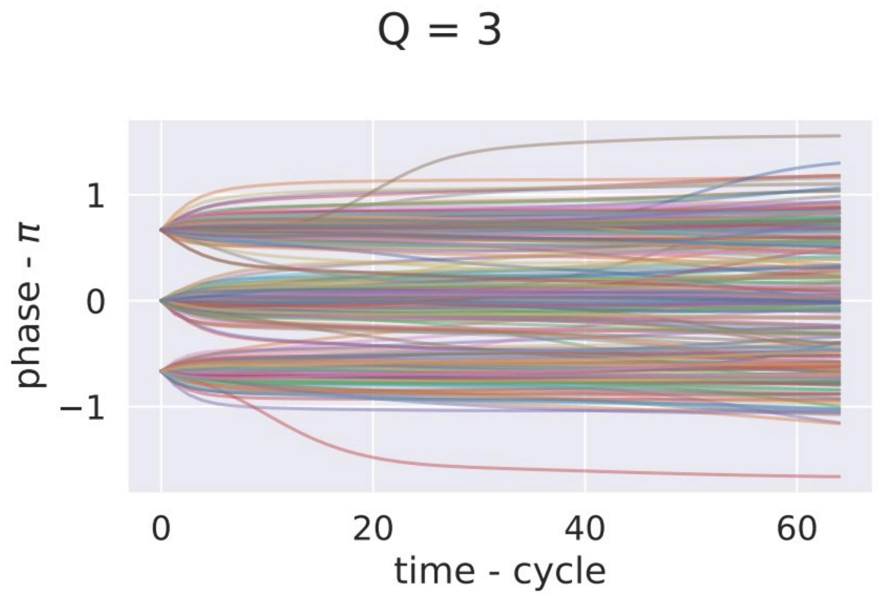
Dynamics

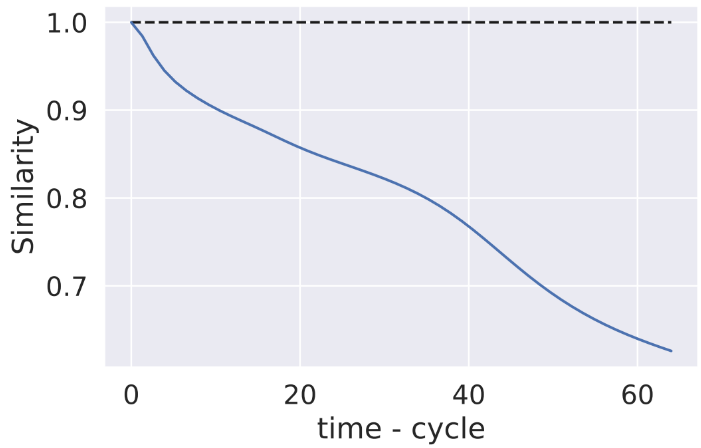
$$\dot{\phi}_i = \epsilon \sum_j R_{ij} \sin(\phi_j + \Phi_{ij} - \phi_i)$$

Parameters

$R_{ij} = |W_{ij}|$ Coupling Strength

$\Phi_{ij} = \arg(W_{ij})$ Coupling Phase Shift





*For M > 2

Phase Quantization

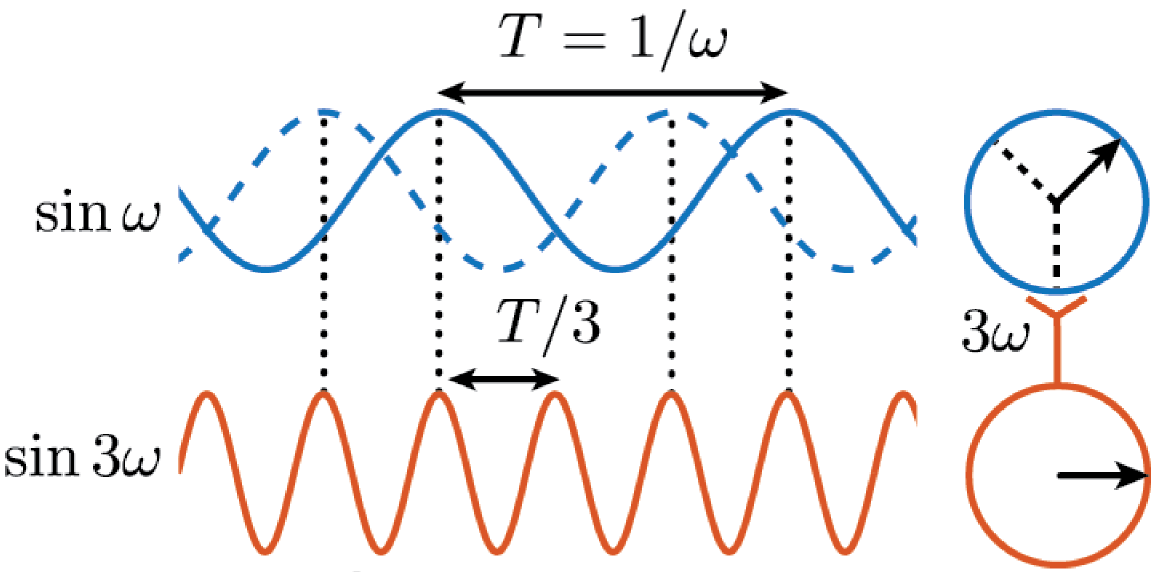
Phase Quantization

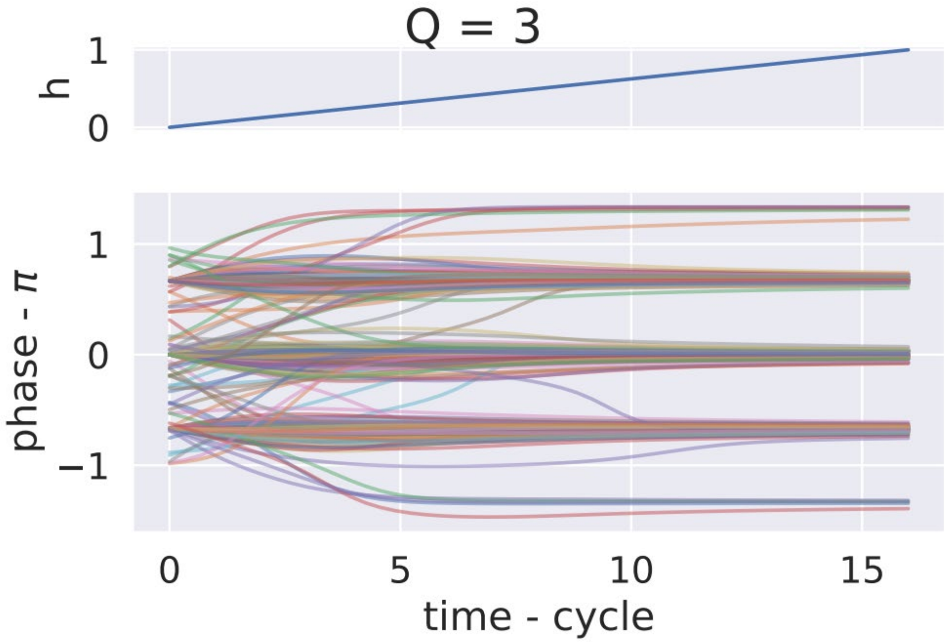
$$f(u_i) = \exp\left(i\frac{2\pi}{Q}\operatorname{argmin}_q\left|\phi_i^{\mathrm{u}} - \frac{2\pi q}{Q}\right|\right)$$

Harmonic Injection Locking (HIL)

$$\dot{\phi}_i = -\epsilon\frac{\partial E(\boldsymbol{\phi})}{\partial \phi_i} - h\sin(Q\phi_i - \phi_j)$$

$$Q = \frac{\omega_{inj}}{\omega_{mem}}$$ Harmonic Ratio



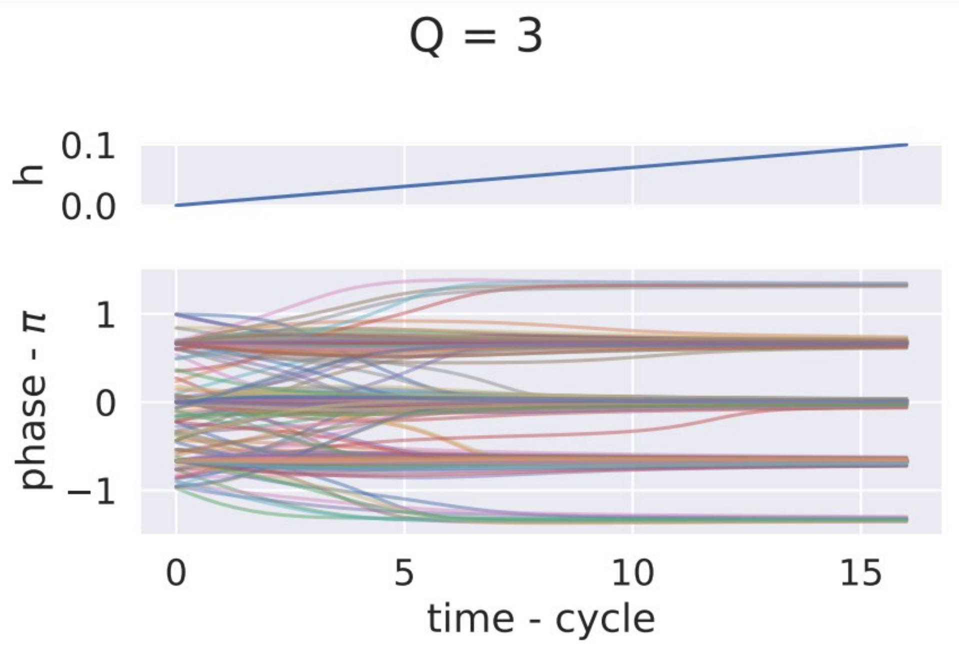


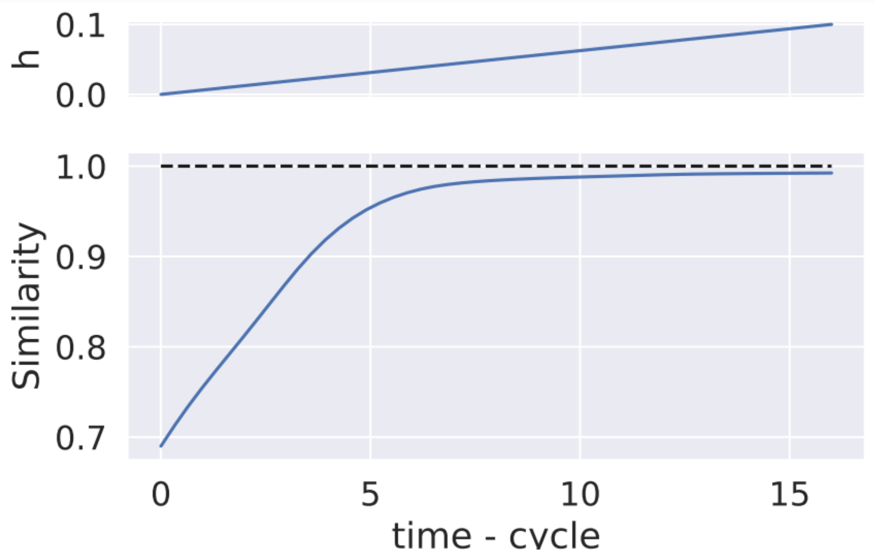
*Presented in Nishikawa et at. '92 for bipolar patterns

Result: Q-State Oscillator Network

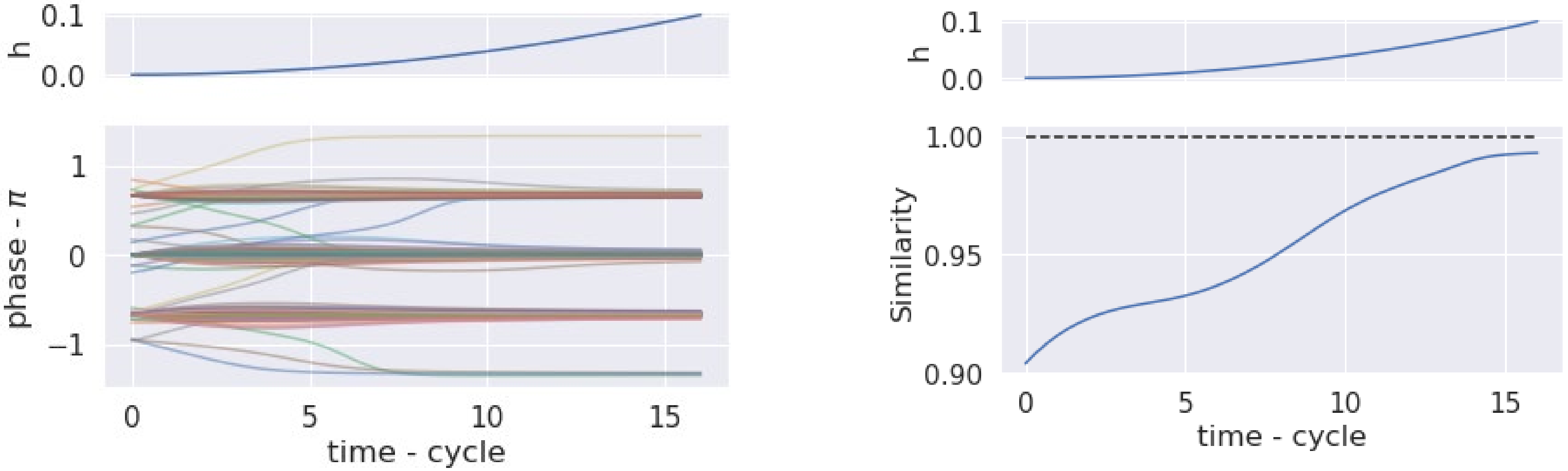
Complete System Dynamics

$$\dot{\phi}_i = \epsilon \sum_j R_{ij} \sin(\phi_j + \Phi_{ij} - \phi_i) - h \sin(Q\phi_i - \phi_j)$$



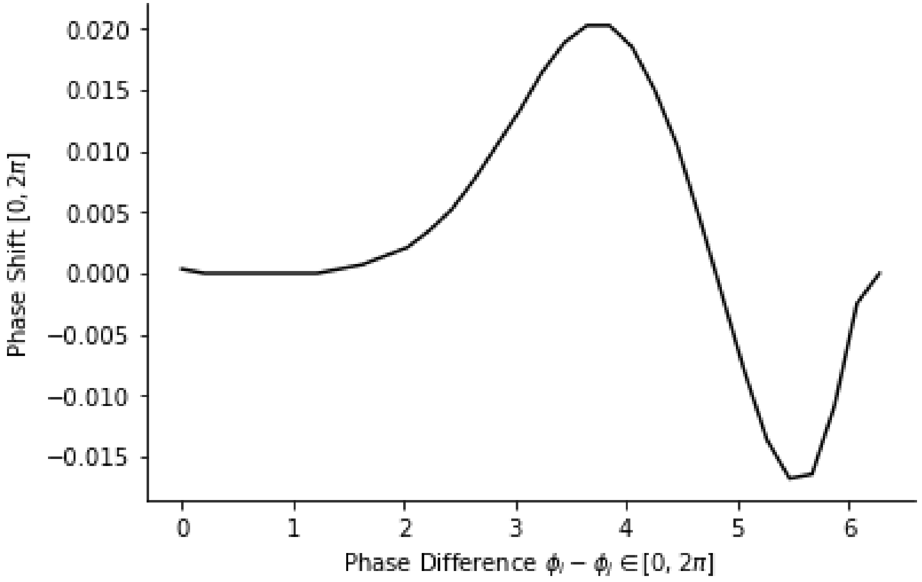


Capacity of Q-state Oscillator Models

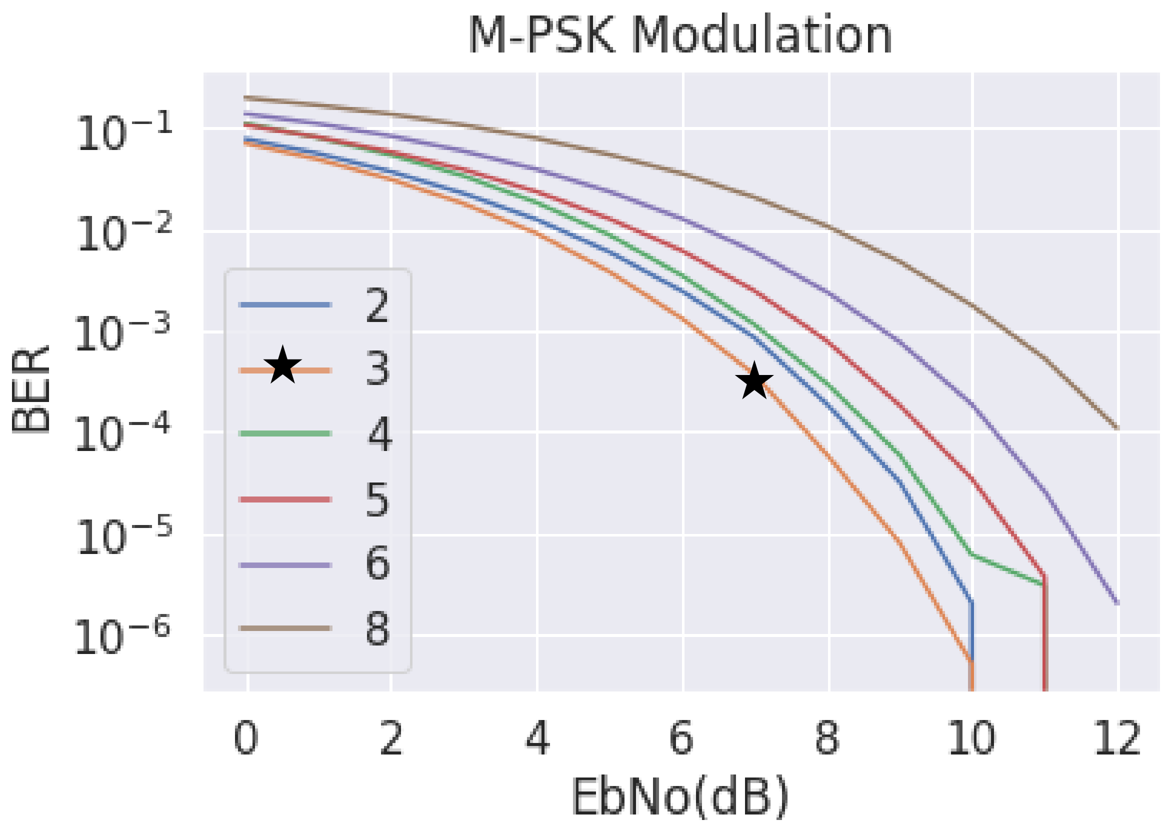


General Phase Coupling

$$\dot{\phi}_i = \sum_j C_{ij} g(\phi_j - \phi_i)$$



M-ary Phase Shift Keying (M-PSK)



★ Again, $Q = 3$ is optimal!!

Summary

Dense Hopfield associative memories in the literature have low capacity

Q-State Phasor Associative Memories achieve high capacity

Implementation of Q-state Phasor Associative Memories in couple oscillators with harmonic injection

$Q = 3$ is best!

Acknowledgments

REDWOOD CENTER
for Theoretical Neuroscience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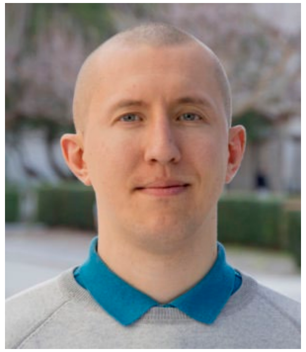
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