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Representations of uncertainty for inference and learning in neural networks

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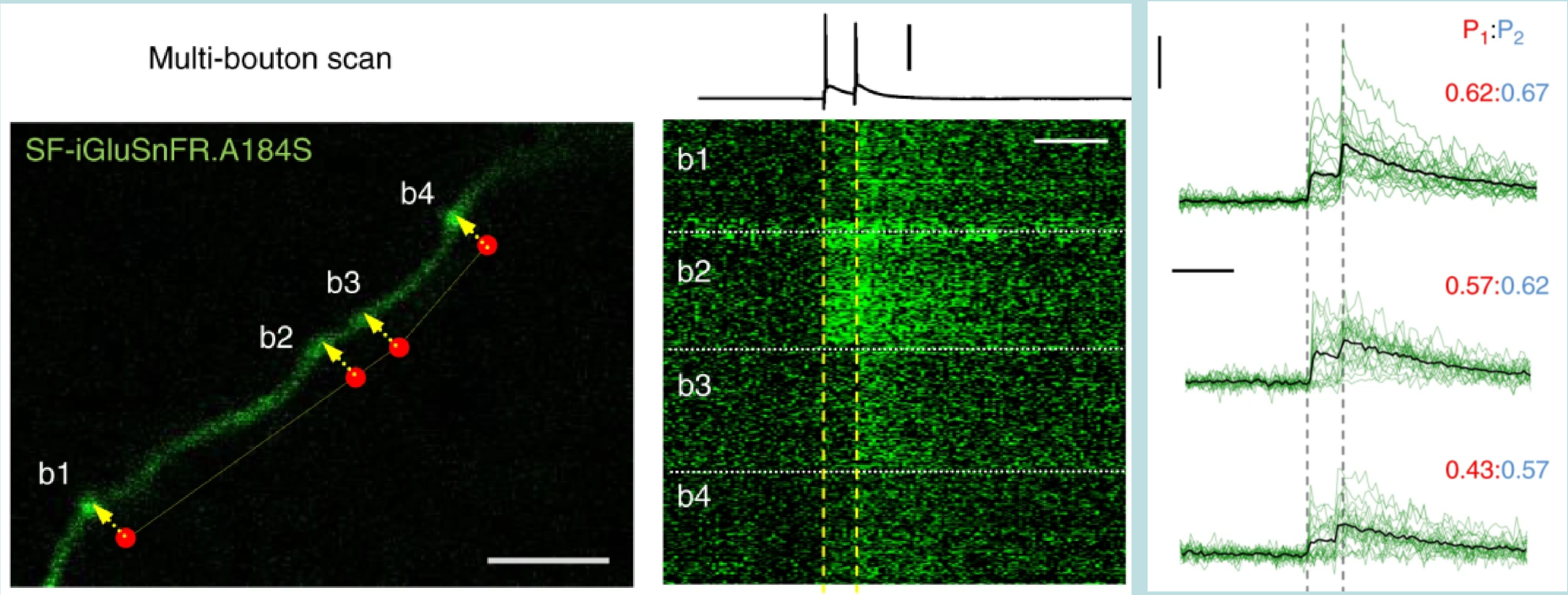
OUTLINE

Representations of uncertainty for inference and learning in neural networks.

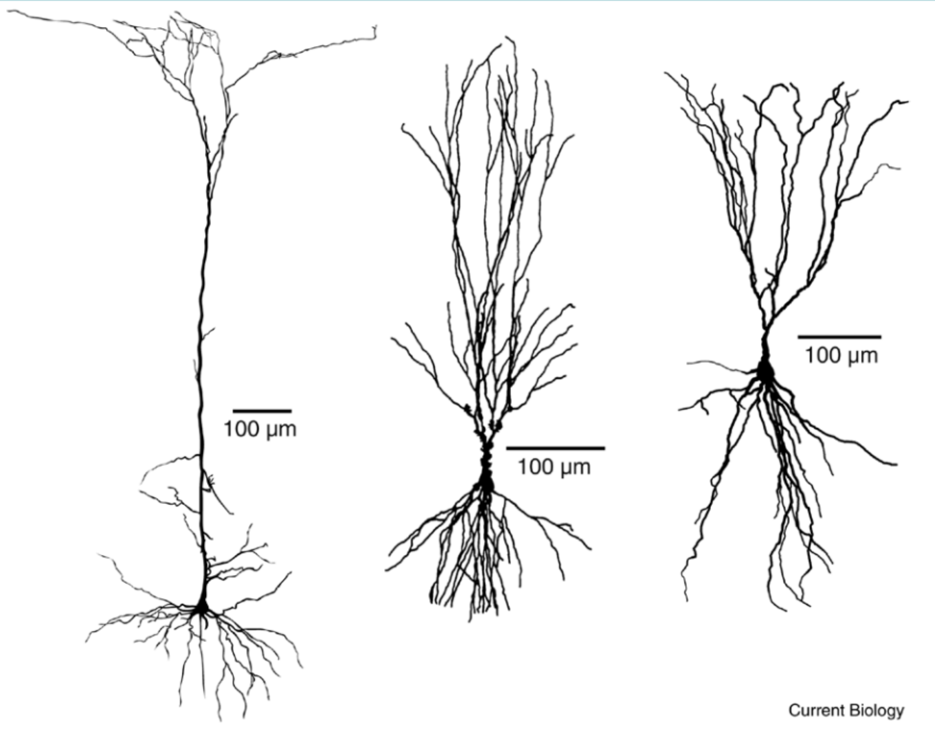
- Single synapses of spiking neural network represent uncertainty through stochastic release.
- Utilizing representations of uncertainty for distributed learning in deep neural network.

Motivation: How can learning be made resilient against - or even benefit from - high levels of **noise in synaptic transmission**?



Trial-to-trial variability of post-synaptic currents in biological synapses [Jensen et al. 2019]

Motivation: How can learning be made resilient against - or even benefit from - high levels of **noise in synaptic transmission**?



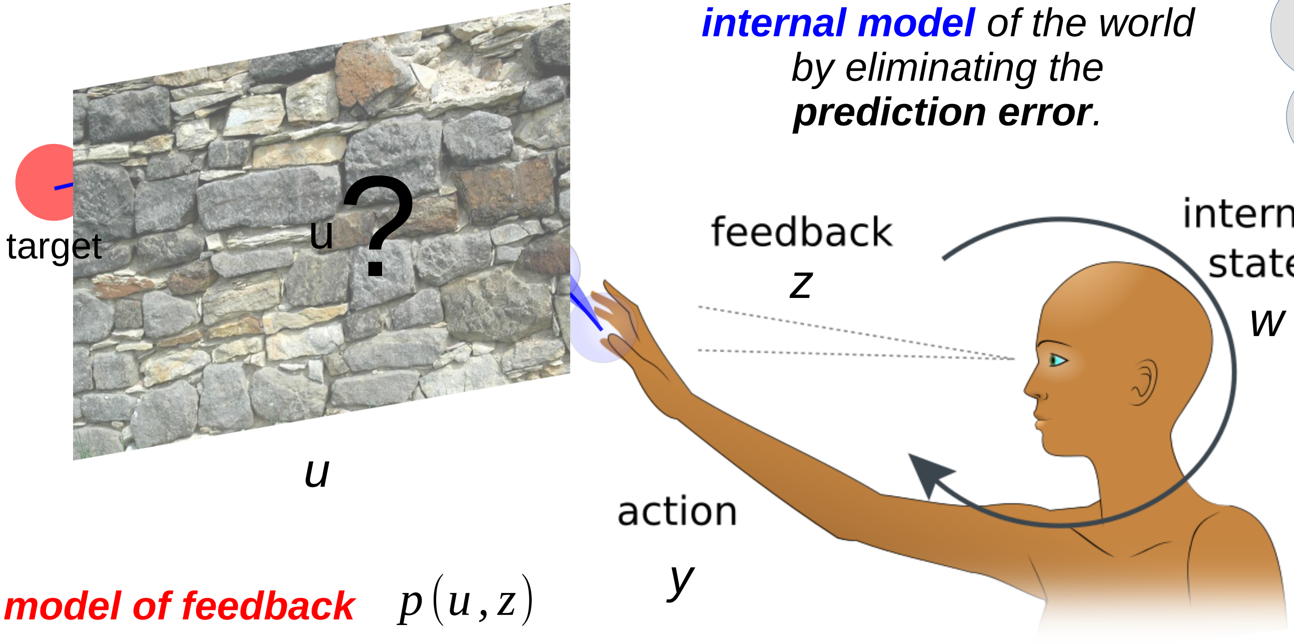
L5 CA3 L2

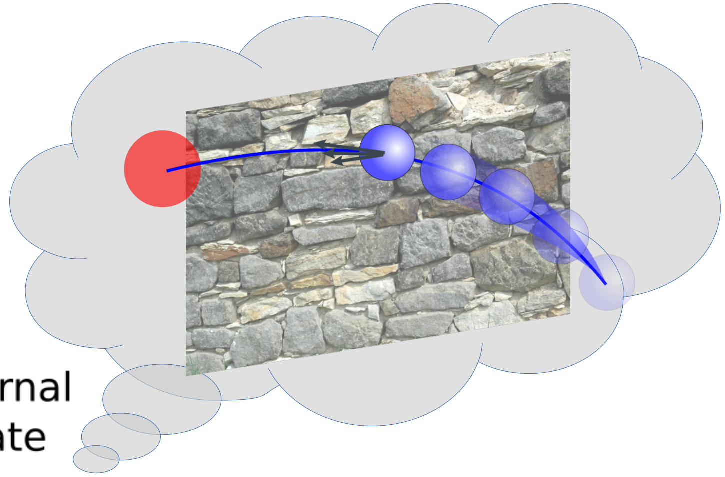
[Bekkers, 2011]

- Synapses are strongly compartmentalized and often experience different voltages than parent dendrites [Cornejo et. al 2022].
- Only the most prominent electrical signals, like back-propagating action potentials, can be detected by synapses located far a way from the soma.
- This suggests a very sparse exchange of information between synapses and parent dendrites.
- **How do synapses cope with these high levels of uncertainty about their environment?**

Free energy model of behavior under uncertainty

*Minimize the mismatch between **feedback** and the **internal model** of the world by eliminating the **prediction error**.*





model of feedback $p(u,z)$

model of internal states $q_w(u)$

$$F(z,w)=\left\langle \log \frac{q_w(u)}{p(u,z)} \right\rangle_{q_w(u)}$$

$$\Delta w = -\frac{\partial}{\partial w} F(z,w)$$

How can learning be made resilient against - or even benefit from - high levels of **noise in synaptic transmission**?

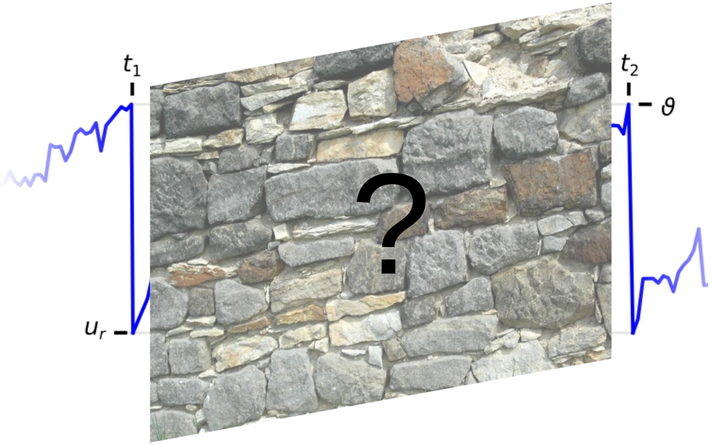
Our hypotheses:

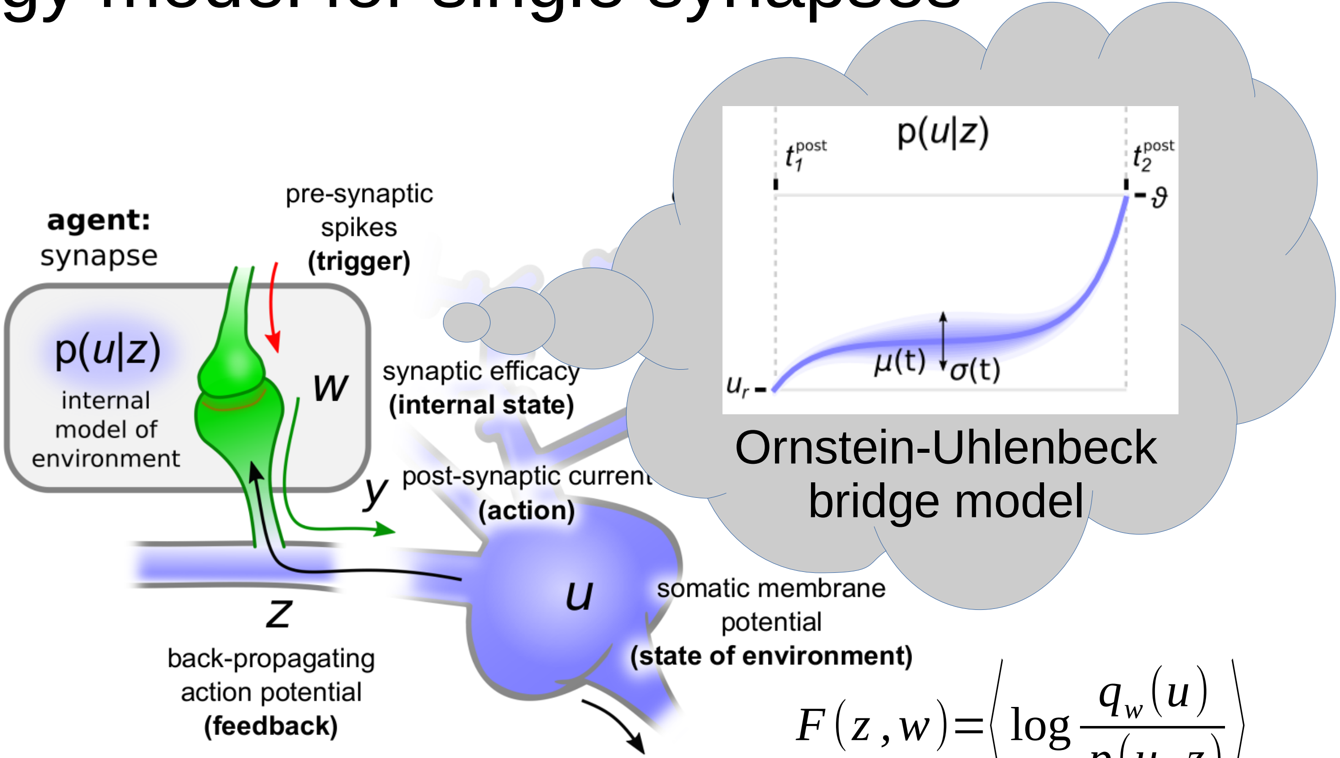
- *The model to act under uncertainty manifests not only on the behavioral level but* ***every synapse*** *utilizes a similar strategy.*

- *Noisy synaptic transmissions are used to* ***express uncertainty*** *about the environment and enable robust learning.*

- *Experimentally observed synaptic plasticity mechanisms are manifestations of these principles for* ***learning to predict post-synaptic activity****.*

Free energy model for single synapses

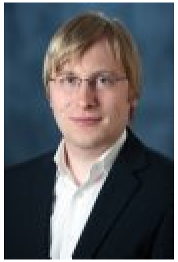




$$F(z,w)=\left\langle \log \frac{q_w(u)}{p(u,z)} \right\rangle_{q_w(u)}$$

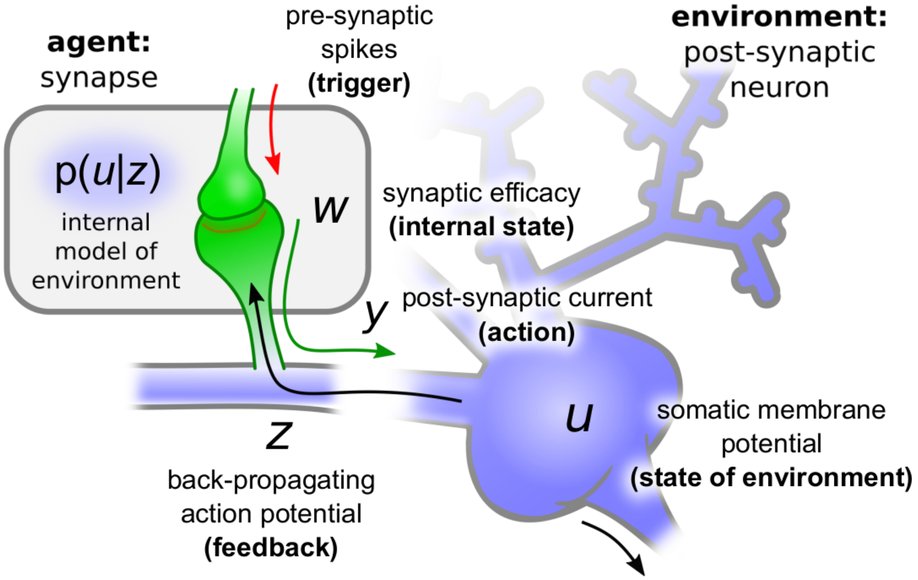
Variational free energy minimization to model the behavior of individual synapses

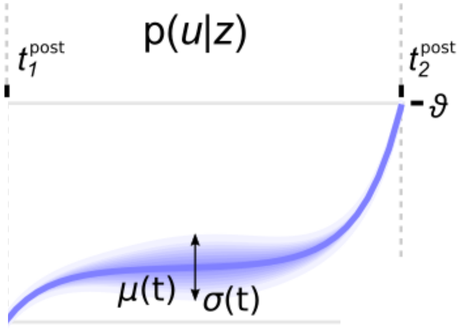
$$\Delta w=-\frac{\partial}{\partial w}F(z,w)$$



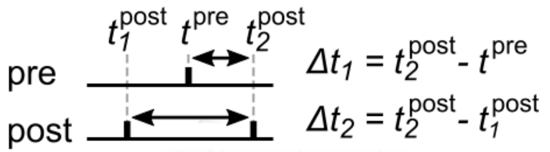
Kappel and Tetzlaff, 2021.

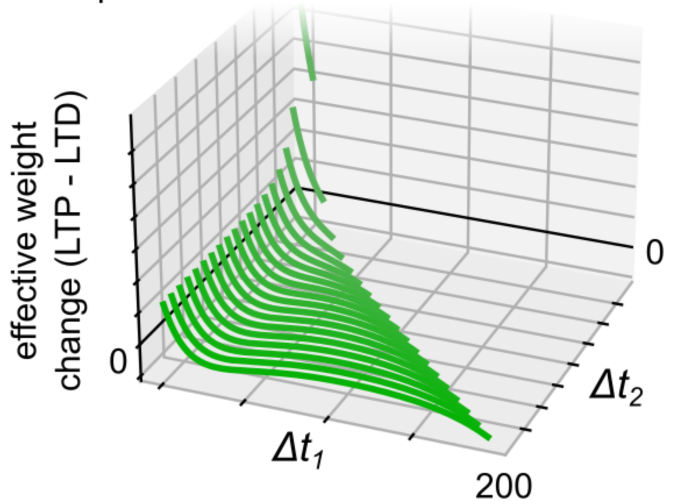
Triplet STDP learning window for free energy minimization





$$\Delta w = -\frac{\partial}{\partial w} F(z, w)$$

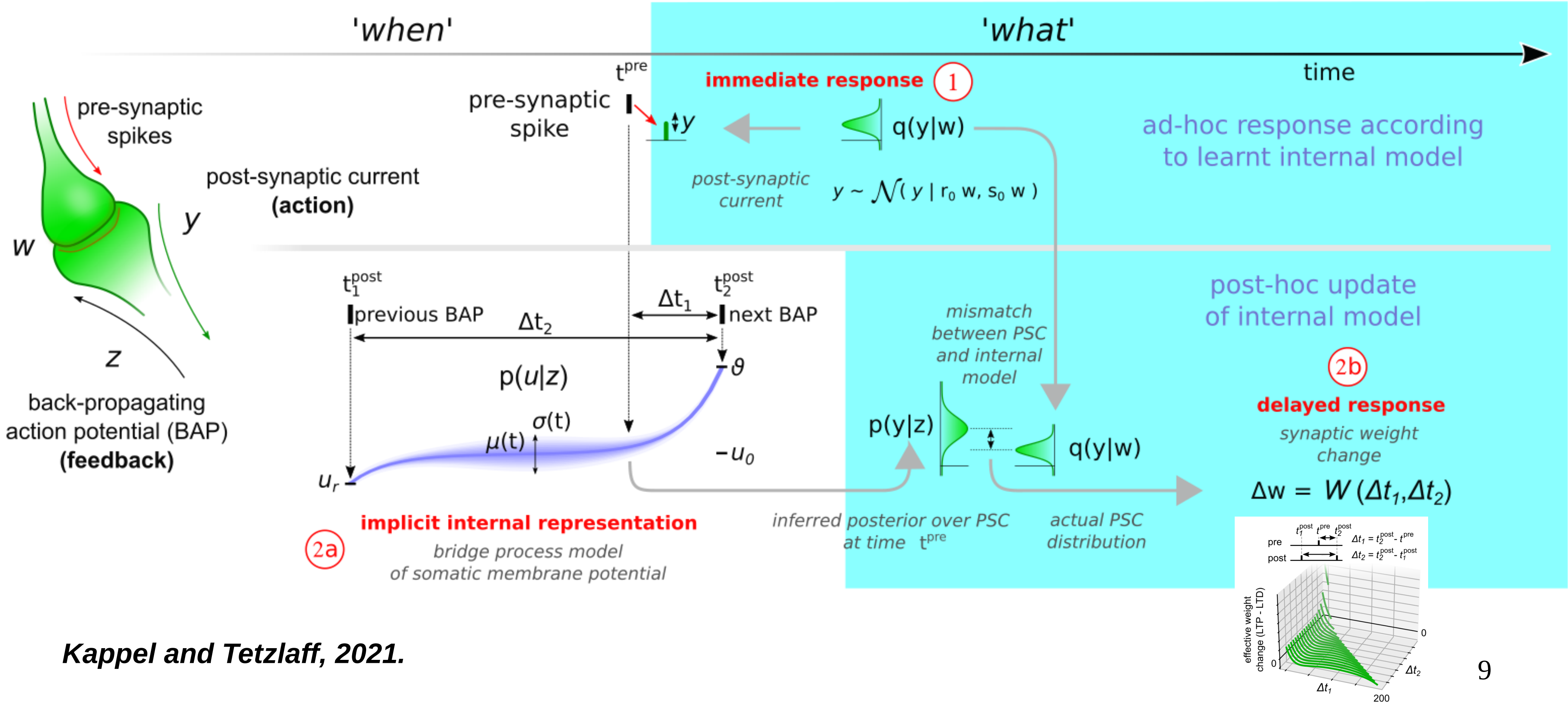




Variational free energy minimization to model the behavior of synapses

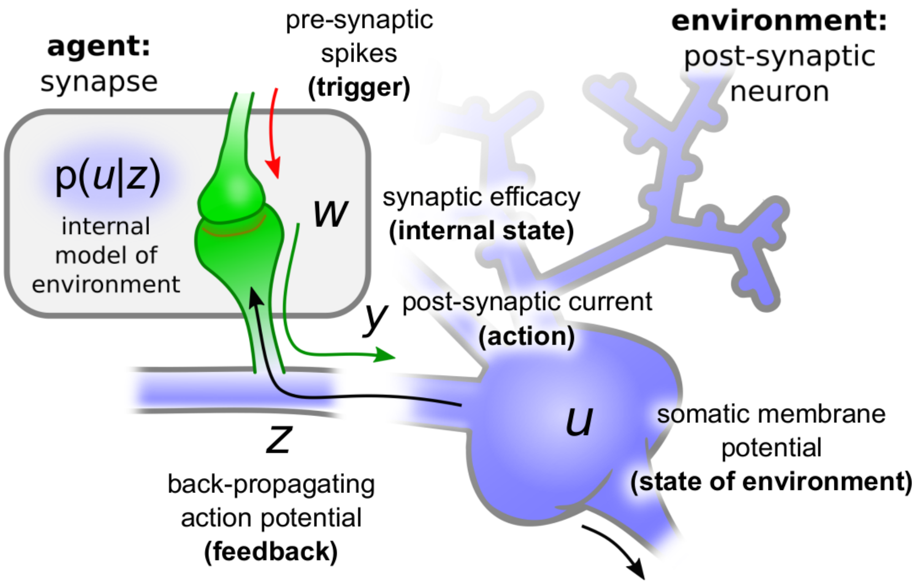
Kappel and Tetzlaff, 2021.

Event-based algorithm for free energy minimization



Kappel and Tetzlaff, 2021.

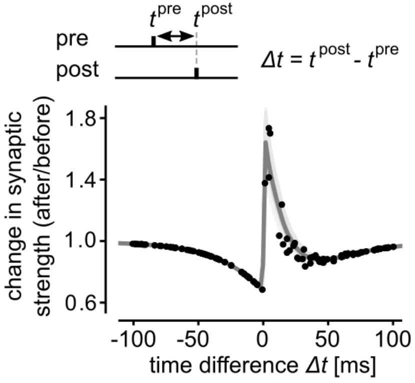
Triplet STDP learning window for free energy minimization



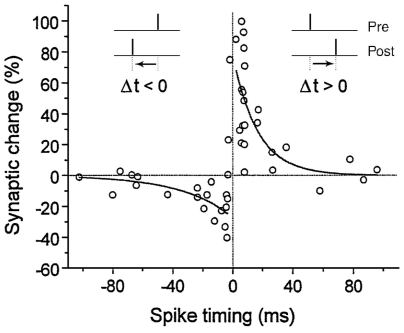
Variational free energy minimization to model the behavior of synapses

Kappel and Tetzlaff, 2021.

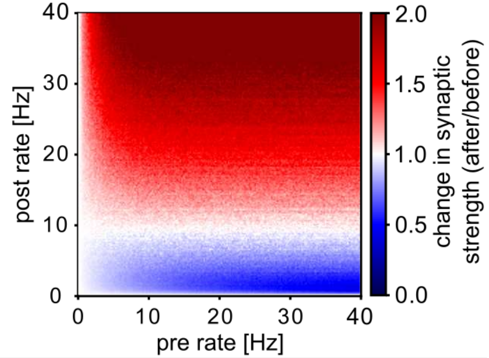
our model



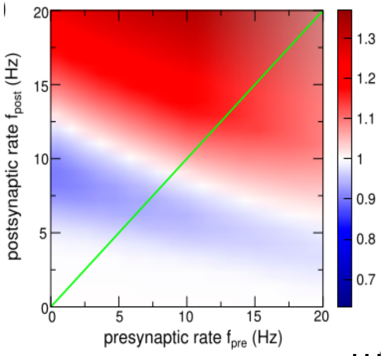
Bi et al. 1998



our model



Graupner and Brunel, 2012



INTERMEDIATE SUMMARY I: FREE ENERGY MODEL OF SYNAPTIC BEHAVIOR

- Single synapses of spiking neural network represent uncertainty through stochastic release.

- Rules to learn synaptic weights to best predict the environment can be derived from first principles and match experimentally found learning dynamics.

- We demonstrate that synapses learn to utilize synaptic noise to represent uncertainties on the level of **single neurons**, **networks for pattern detection**, and **closed loop behavior**
  → Kappel and Tetzlaff 2022, bioarxiv.rog/2022.04.22.489175.full.pdf

OUTLINE

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- Single synapses of spiking neural network represent uncertainty through stochastic release.
- Utilizing representations of uncertainty for distributed learning in deep neural network.

Representing uncertainties in deep neural networks

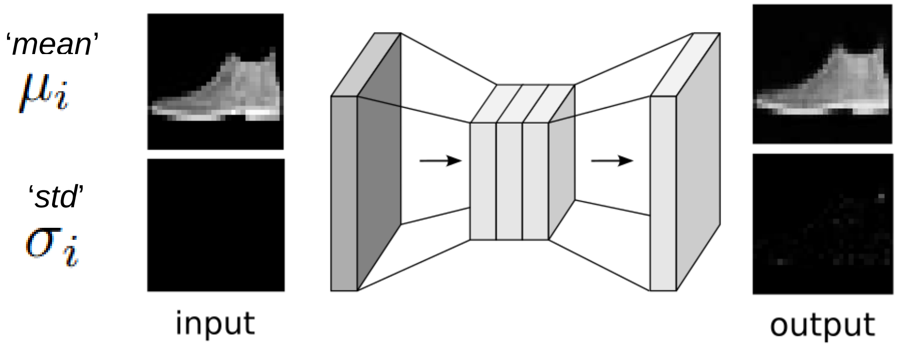
- Representing uncertainties in terms of probabilities is already implicit in many methods that are commonly used in deep learning, e.g. cross entropy loss, mean squared error.

- However few deep learning models make explicit use of these uncertainty representations.

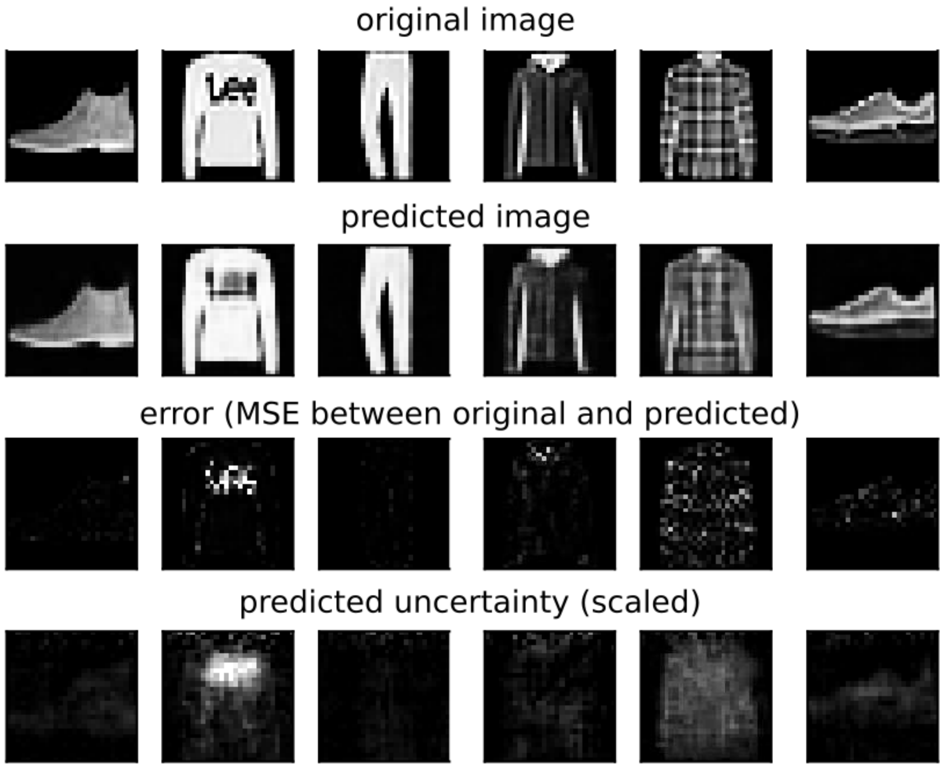
- We suggest to introduce uncertainty representations explicitly into DNNs by interpreting the output of a DNN as the parameters of a probability distribution *q(x)* and then use probability theory to *derive learning rules* to learn a target probability distribution *p(x).*

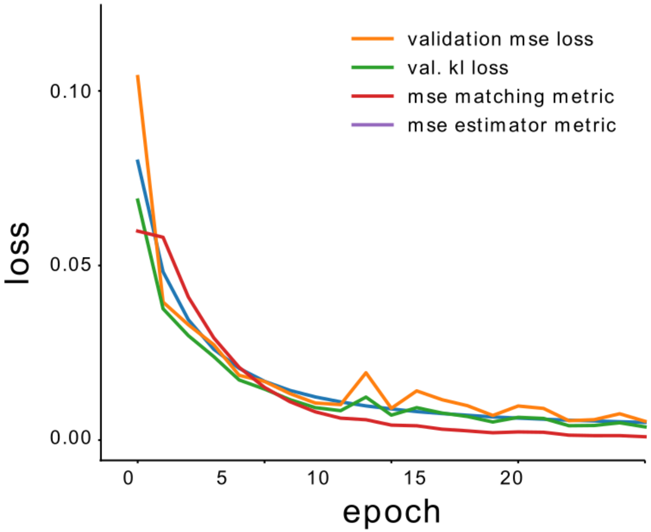
$$D_{KL}(p, q) = \int p(x) \log \frac{p(x)}{q(x)} dx$$

Example: Uncertainty-aware autoencoder

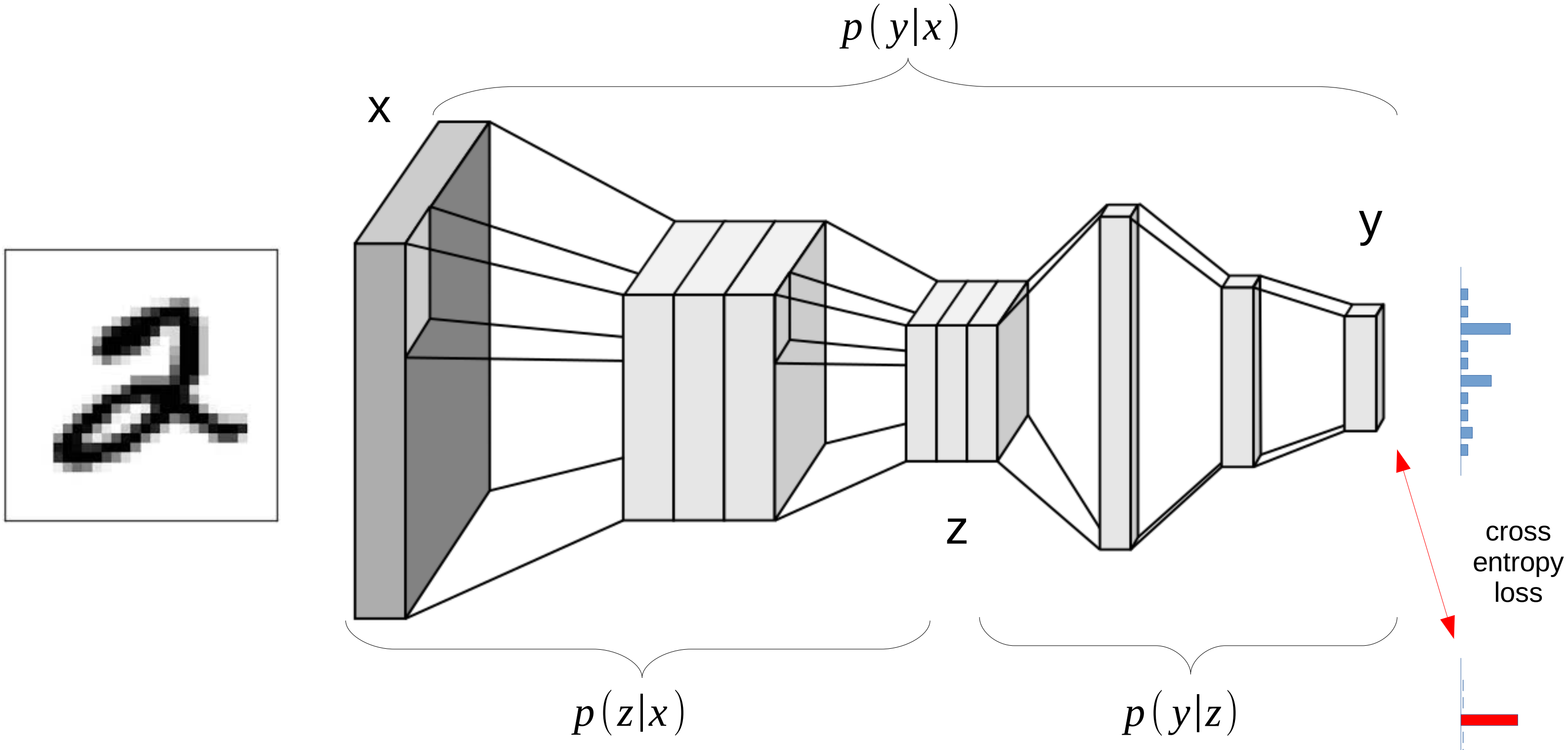


$$\mathcal{D}_{KL}\left(p\,|\,q\right) = \log\frac{\sigma_i}{\sigma_i^*} + \frac{1}{2}\left(\frac{\sigma_i^2}{\left(\sigma_i^*\right)^2} + \frac{\left(\mu_i - \mu_i^*\right)^2}{\left(\sigma_i\right)^2} - 1\right)$$





Using uncertainties for distributing learning

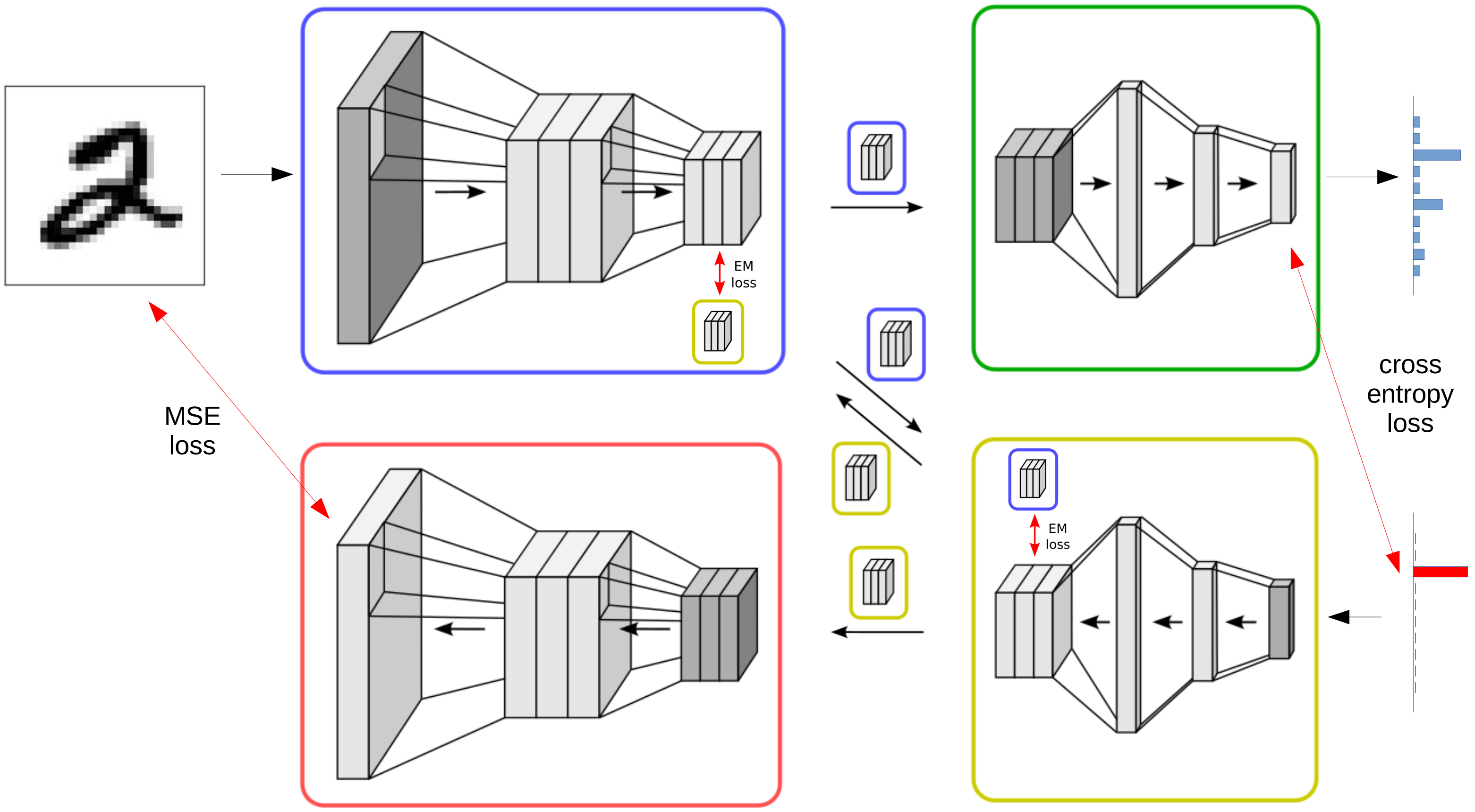


$$p(y|x) = \int p(y|z)\, p(z|x)\, dz$$

$$\frac{\partial}{\partial \theta} \log p(y|x) = \int p(z|x, y) \left(\frac{\partial}{\partial \theta} \log p(z|x) + \frac{\partial}{\partial \theta} \log p(y|z) \right) dz$$

EM-like
decomposition

First PoC Application: “LeNet” (-like) architecture



test accuracy:

98.93%
(baseline end-to-end with MSE loss)

98.96%
(end-to-end learning with EM loss)

98.26%
(with EM loss and stop gradient)

14.21%
(w/o EM loss but with stop gradient)

INTERMEDIATE SUMMARY II: DISTRIBUTED LEARNING USING UNCERTAINTIES

- DNNs are surprisingly good in generalizing in probability spaces.
- We suggest to exploit this property to utilize uncertainties for distributed learning.
- Distribute global learning across a network with nodes that use only local learning → uncertainty messages to solve the credit assignment problem.
- Mechanism for failure prediction automatically built in → might come in handy in all sorts of applications.