

```
tau_minus = 33.7e-3;  
tau_long = 114e-3;  
  
tau_homeostatic = tau_hom;  
  
/* Initialization of presynaptic traces */  
tr_pre = src->get_pre_trace(tau_plus);  
  
/* Initialization of postsynaptic traces */  
tr_post = dst->get_post_trace(tau_minus);  
tr_post2 = dst->get_post_trace(tau_long);  
tr_post_hom = dst->get_post_trace(tau_hom);  
  
hom_fudge = A3_plus*tau_plus*tau_long/(tau_min  
  
/* Set min/max weight */  
set_min_weight(0.0);  
set_max_weight(maxweight);  
  
stdp_active = true;  
  
0.2 0.5 0.6 0.8 1.0  
} t
```

```
TripletConnection.cpp  
g_nmda->saxpy(-mul_nmda, g_nmda);  
  
// decay of nmda and gaba channels, i.e. multi  
scale_ampa);  
scale_gaba);  
  
internal state  
I apply the Euler  
egrate membrane() T
```



Affiliated Institute of the University of Basel

Learning algorithms for spiking and physical neural networks

Friedemann Zenke

Computational Neuroscience @ FMI

www.zenkelab.org



Affiliated with Novartis Biomedical Research

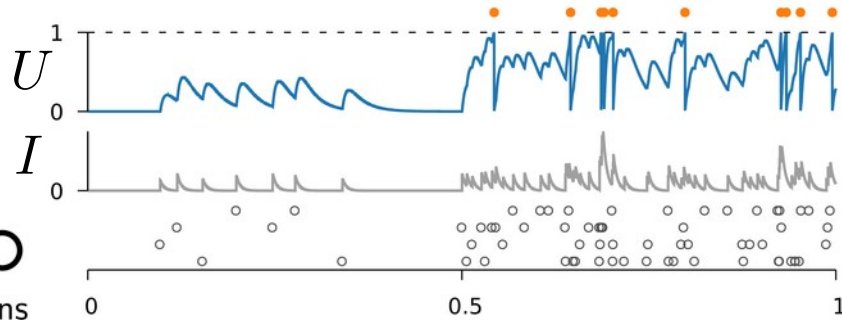
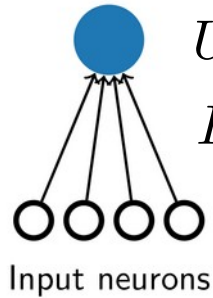
Today: Overview recent work on learning algorithms

- Surrogate gradients for spiking neural networks
 - Getting spiking neural networks to do something interesting (recap)
Neftci, Mostafa, and Zenke (2019) *IEEE SPM*
 - Sidestepping device mismatch on analog neuromorphic substrates
Cramer, B., Billaudelle, S., Kanya, S., Leibfried, A., Grübl, A., Karasenko, V., Pehle, C., Schreiber, K., Stradmann, Y., Weis, J., Schemmel, J., and Zenke, F. (2022). *PNAS*
- Resurrecting local learning rules (beyond backprop)
 - Training noisy substrates with holomorphic equilibrium propagation
Laborieux and Zenke (2022) *Neurips*
 - Online self-supervised learning with local learning rules
Halvagal and Zenke (2023) *Nature Neuroscience*

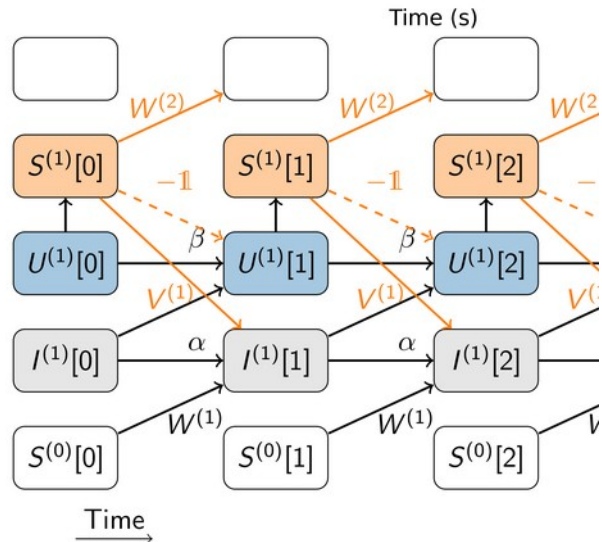
How do we get spiking neural networks to do something interesting?

Through end-to-end training!

Recap: Training spiking networks end-to-end



- Spiking neurons & networks are RNNs
- They have implicit and explicit recurrence
- Known training procedures for networks **with hidden units**
 - Backpropagation-through time (BPTT)
 - Real-time recurrent learning (RTRL)



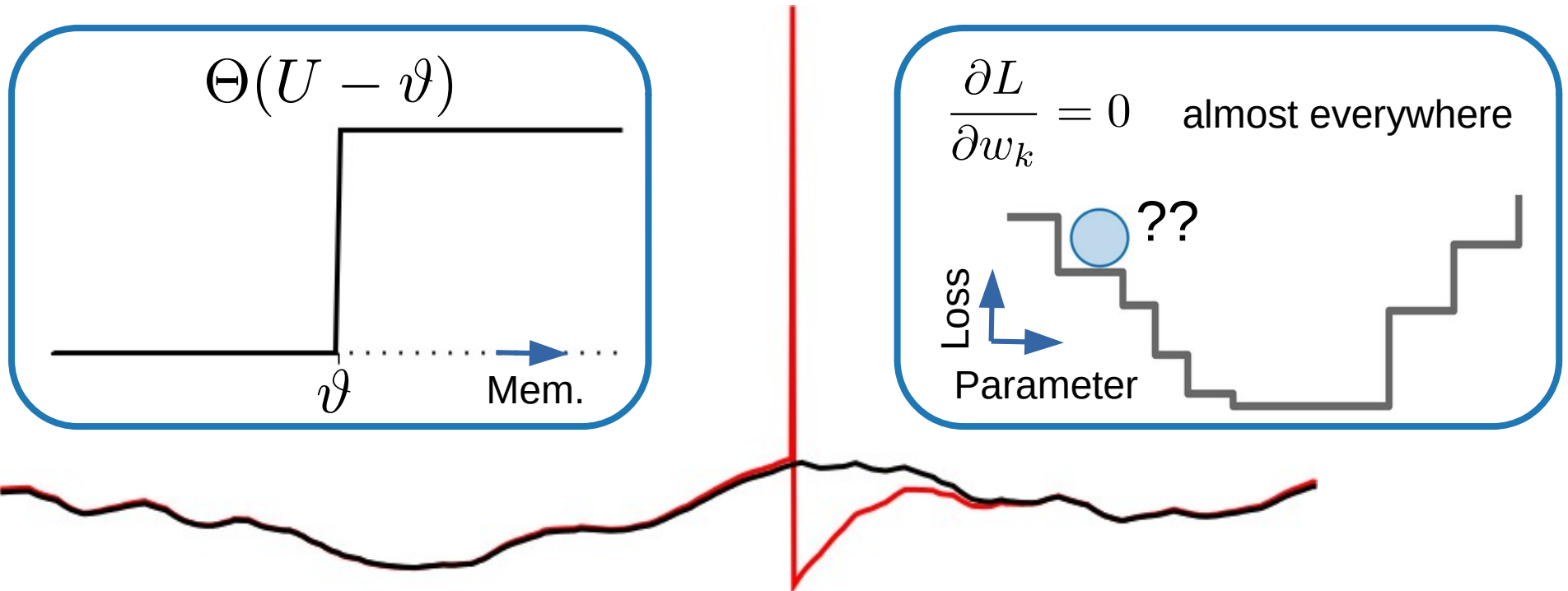
$$S_i^{(1)}[n] = \Theta \left(U_i^{(1)}[n] - \vartheta \right) \quad \text{Problem}$$

$$U_i^{(1)}[n + 1] = \beta U_i^{(1)}[n] + I_i^{(1)}[n] - S_i[n]$$

$$I_i^{(1)}[n + 1] = \underbrace{\alpha I_i^{(1)}[n]}_{\text{exp. current decay}} + \underbrace{\sum_j W_{ij} S_j^{(0)}[n]}_{\text{feed-forward input}}$$

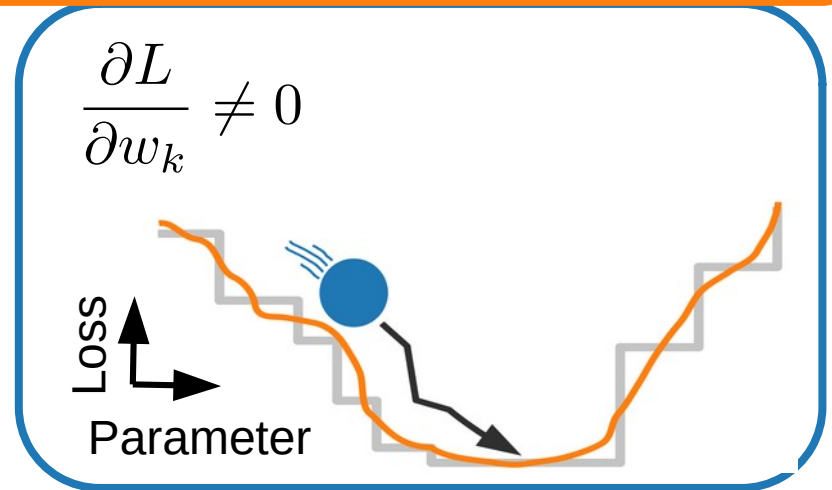
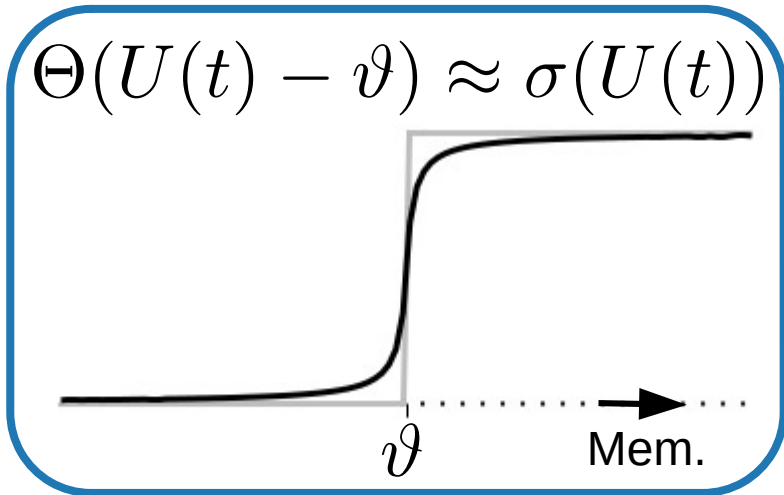
Forward Euler integration

Problem: The derivative of a spike train is zero almost everywhere



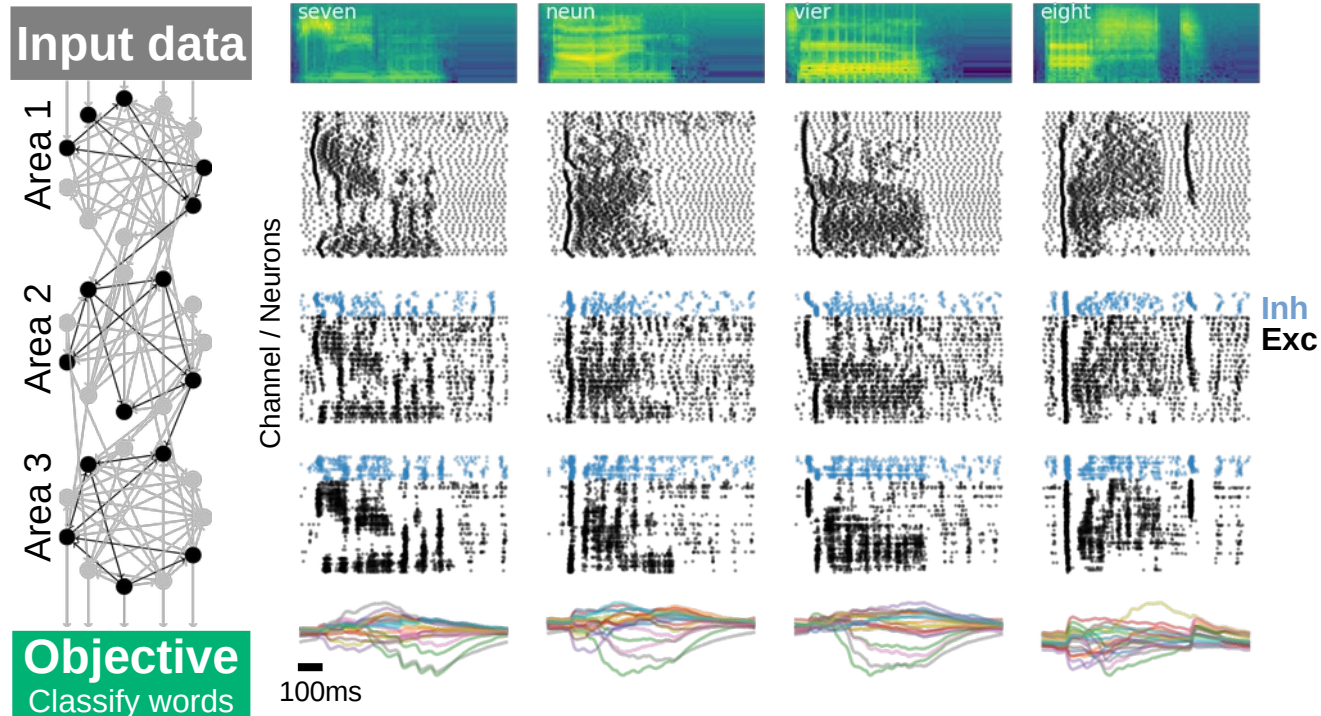
Solution: Surrogate gradient

Bohte (2011), Esser et al. (2015), Bellec et al. (2018), Shrestha & Orchard (2018), Zenke & Ganguli (2018), ...
In ML: "Straight-through estimators" Bengio et al. (2013)



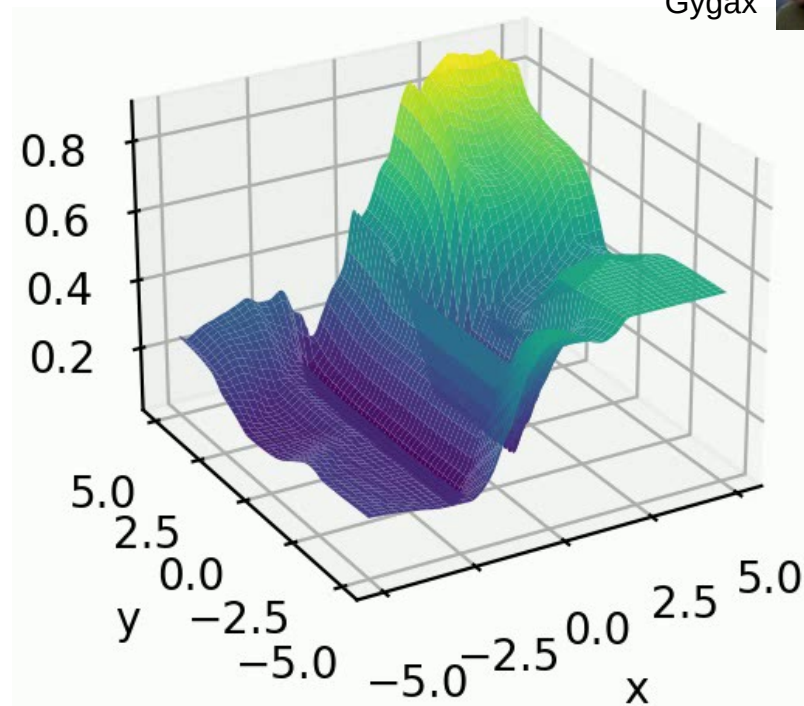
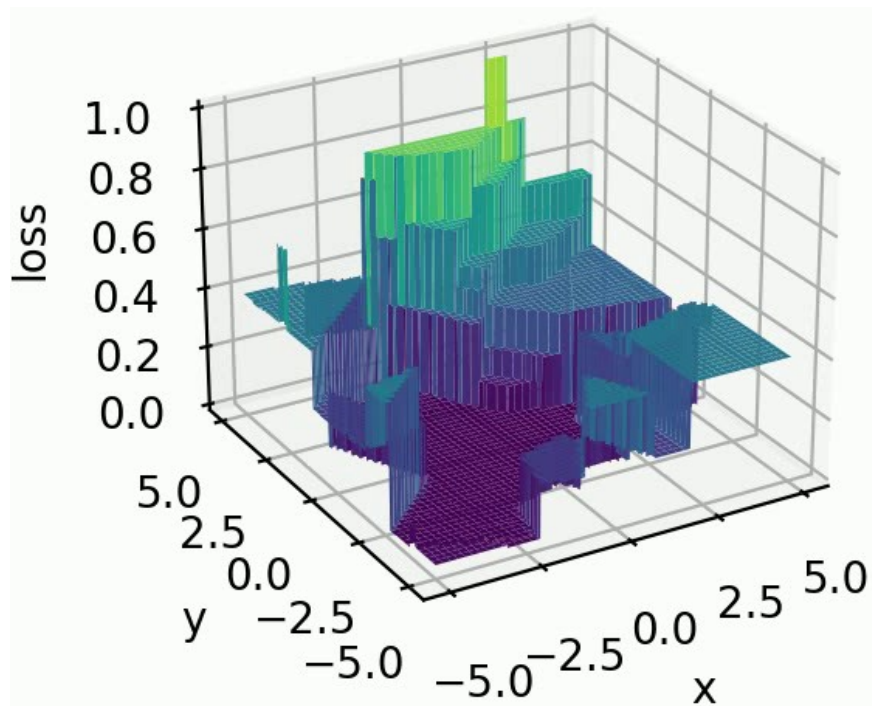
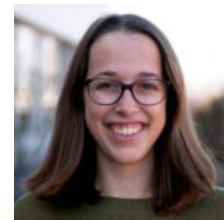
Surrogate gradients

No assumption about rate or time coding required.



Loss landscape of a spiking net (2D projection)

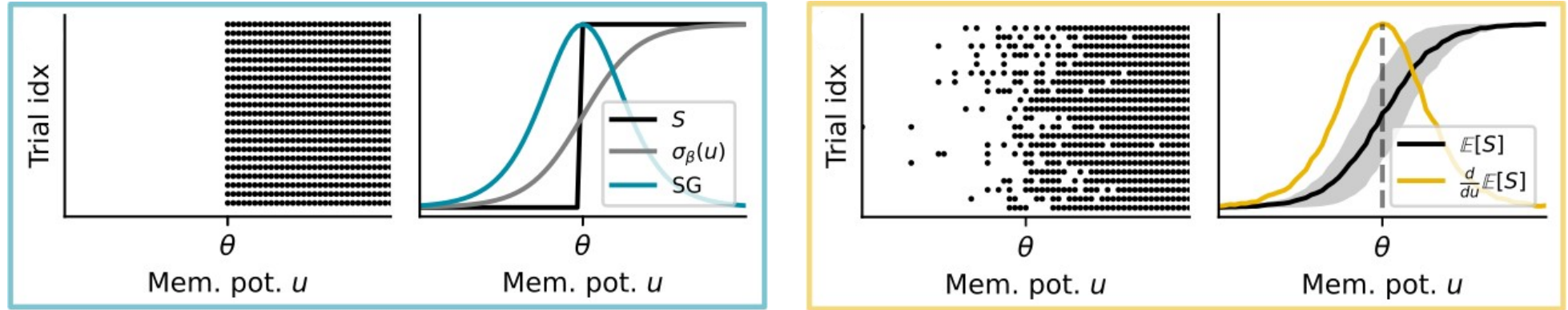
Julia Gyga



Integrated surrogate gradient

Problem: Surrogate gradients are a heuristic and lack theory.

Surrogate gradients are related to well-defined gradients in expectation in single stochastic neurons



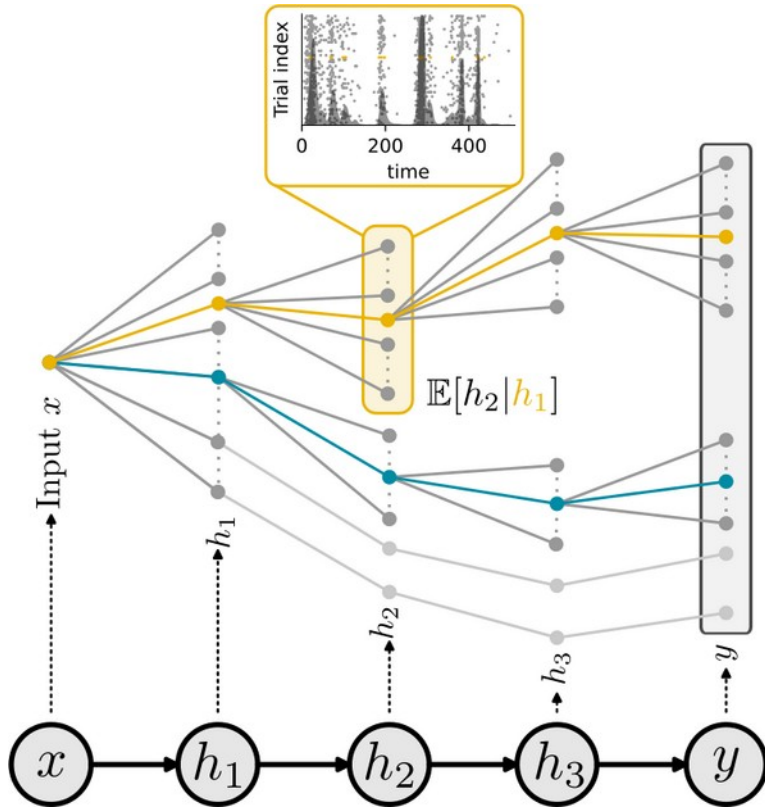
In spiking neurons: Pfister, Toyoizumi, Barber & Gerstner (2006), Gardner, Sporea & Grüning (2015)

But does not work in multi-layer networks because it breaks the chain rule:

$$\mathbb{E} \left[\frac{\partial}{\partial p_y} y \frac{\partial}{\partial h_2} p_y \frac{\partial}{\partial p_2} h_2 \dots \frac{\partial}{\partial w_1} p_1 \right] \neq \mathbb{E} \left[\frac{\partial}{\partial p_y} y \right] \mathbb{E} \left[\frac{\partial}{\partial h_2} p_y \right] \mathbb{E} \left[\frac{\partial}{\partial p_2} h_2 \right] \dots \mathbb{E} \left[\frac{\partial}{\partial w_1} p_1 \right]$$

Stochastic Automatic Differentiation provides missing theoretical foundation for surrogate gradients

Julia Gygax



- Finite differences? Does not scale, high variance → not an option

- “Stochastic automatic differentiation”

Arya, Schauer, Schäfer, Rackauckas, 2022. *Neurips*

$$\frac{\partial}{\partial w_1} \mathbb{E}[y] \approx \frac{\partial}{\partial h_2} \mathbb{E}[y|h_2^*] \frac{\partial}{\partial h_1} \mathbb{E}[h_2|h_1^*] \frac{\partial}{\partial w_1} \mathbb{E}[h_1|x]$$

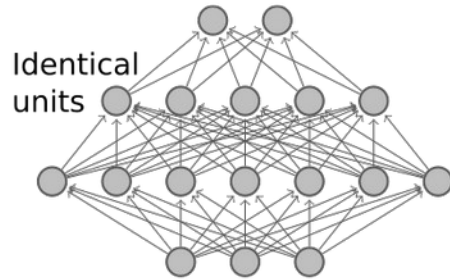
- Surrogate gradients fall out of this framework

How do we perform efficient inference with
(spiking) neural networks?

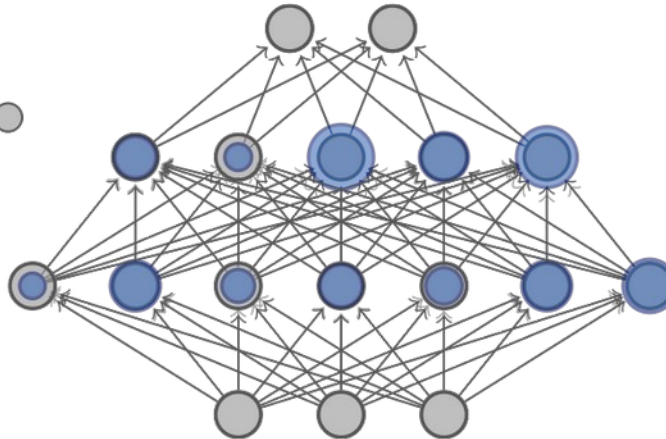
With ultra-low power neuromorphic hardware!
(use the device physics)

Problem: Device mismatch

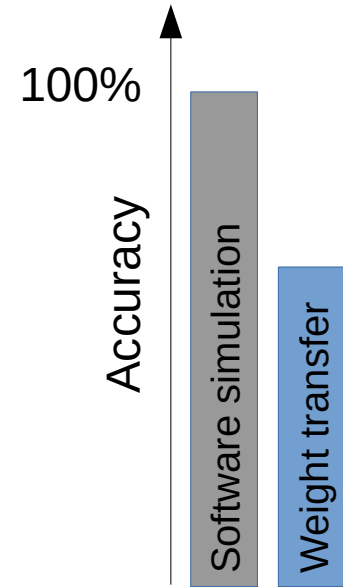
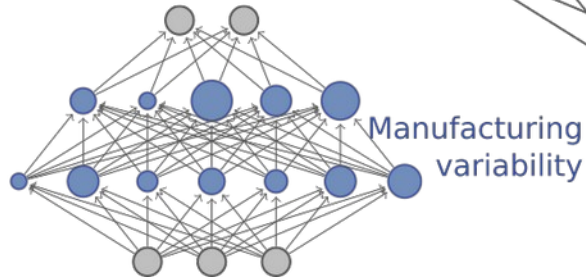
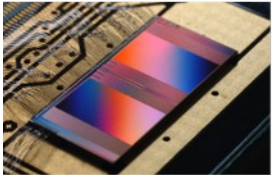
Software implementation



Mismatch!

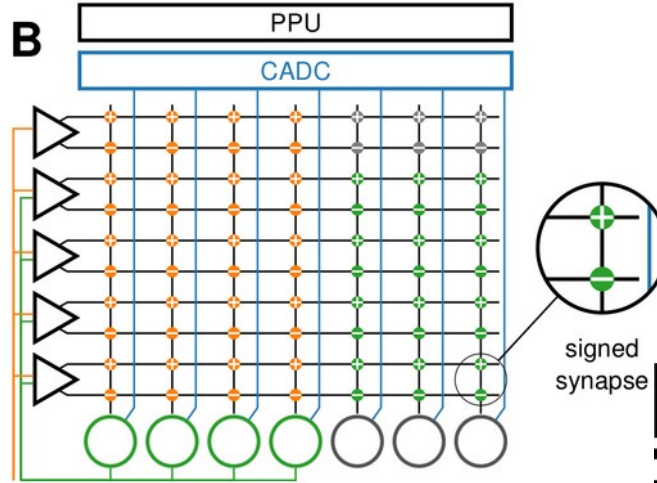
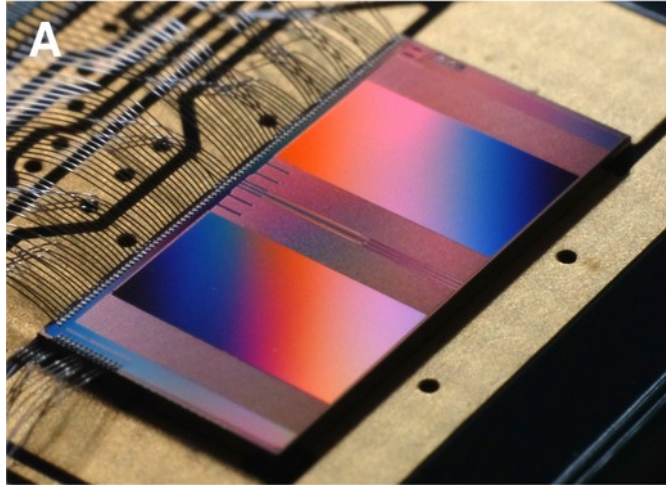


Hardware implementation



→ costly calibration

To study this question we used the BrainScaleS-2 analog neuromorphic hardware system



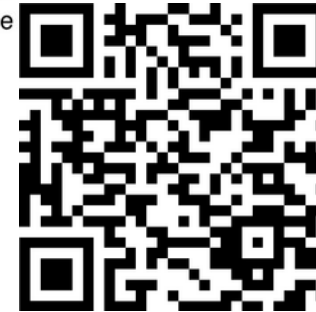
Johannes Schemmel
Uni Heidelberg



Benjamin
Cramer

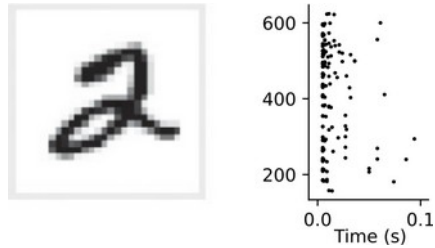


Sebastian
Billaudelle

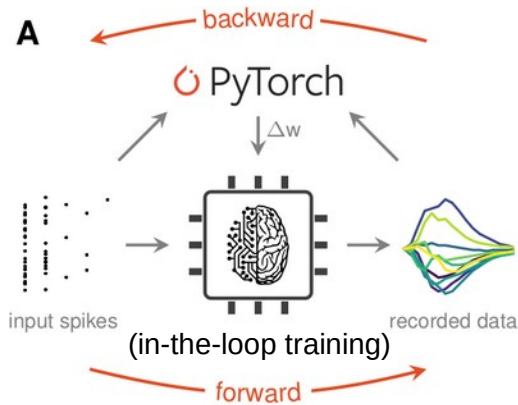
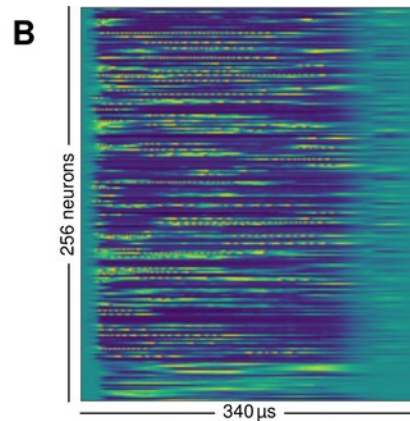


In-the-loop surrogate gradient training

Forward-pass on chip and backward pass in software

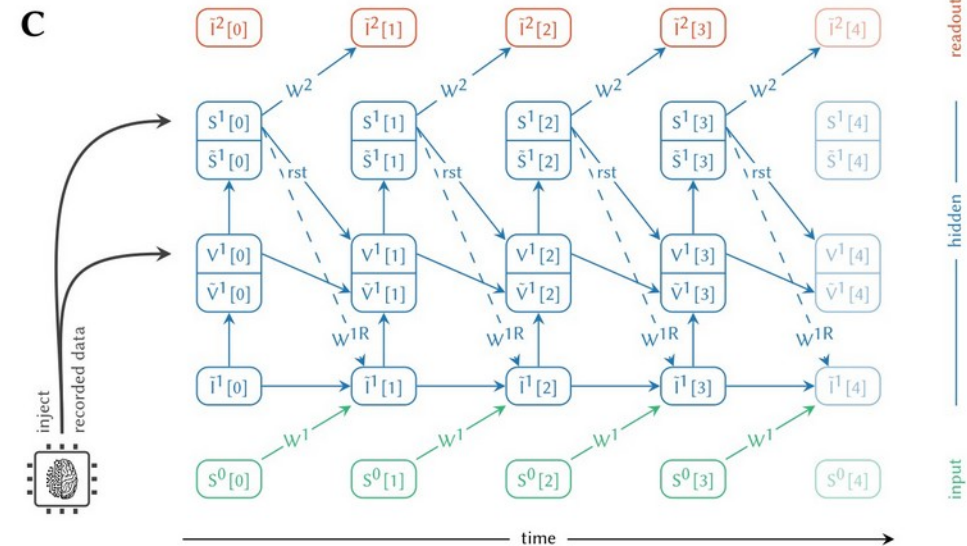


2) Measure on-chip analog voltage traces



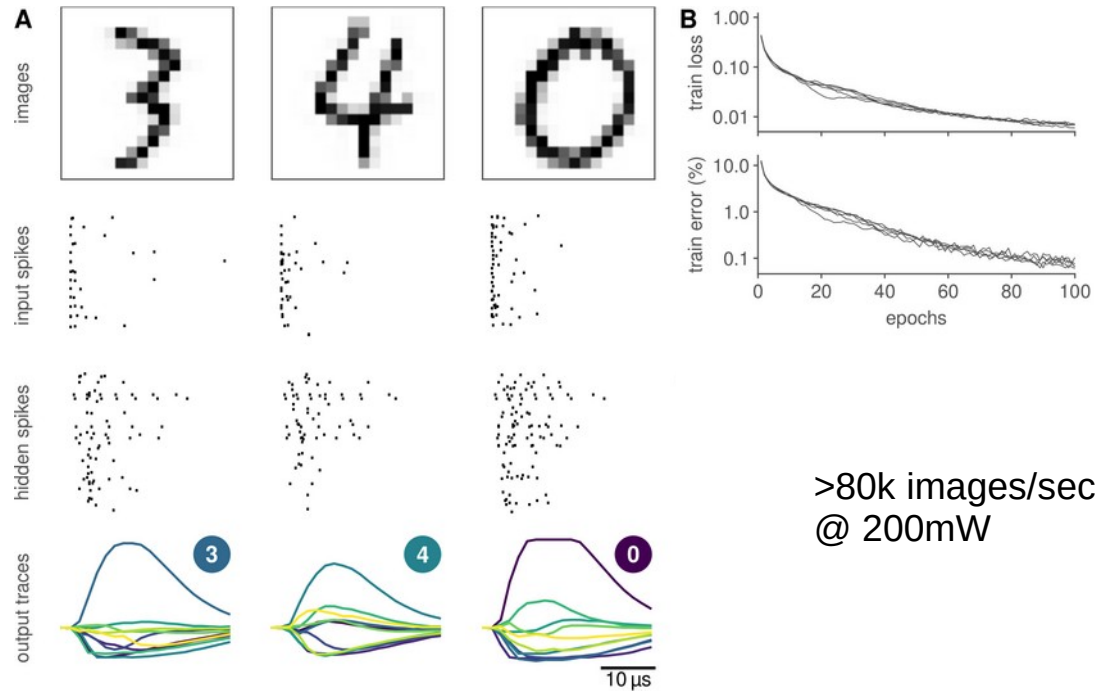
1) Forward pass on chip

3) Inject voltage into computational graph

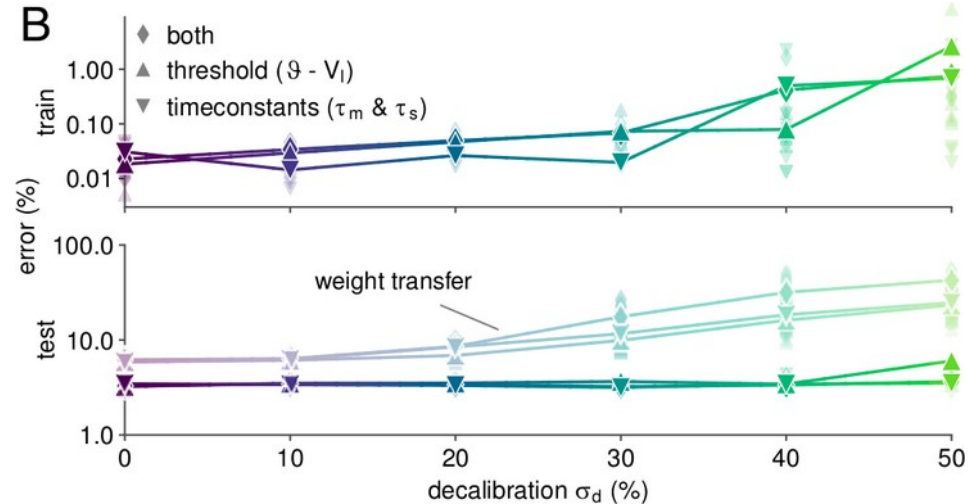
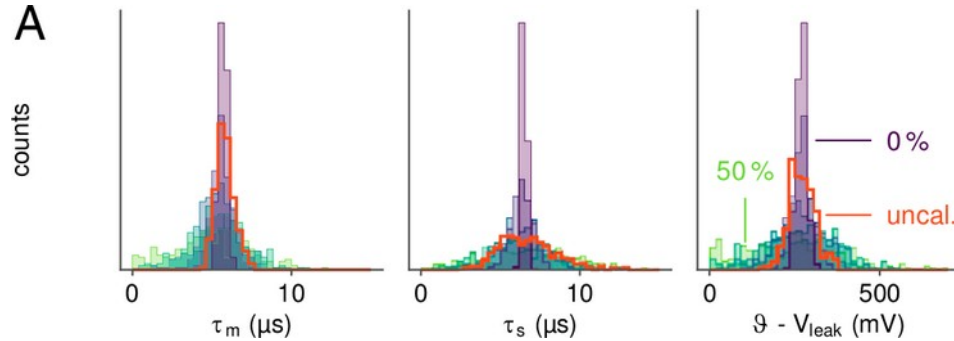


4) Compute surrogate gradients → update weights

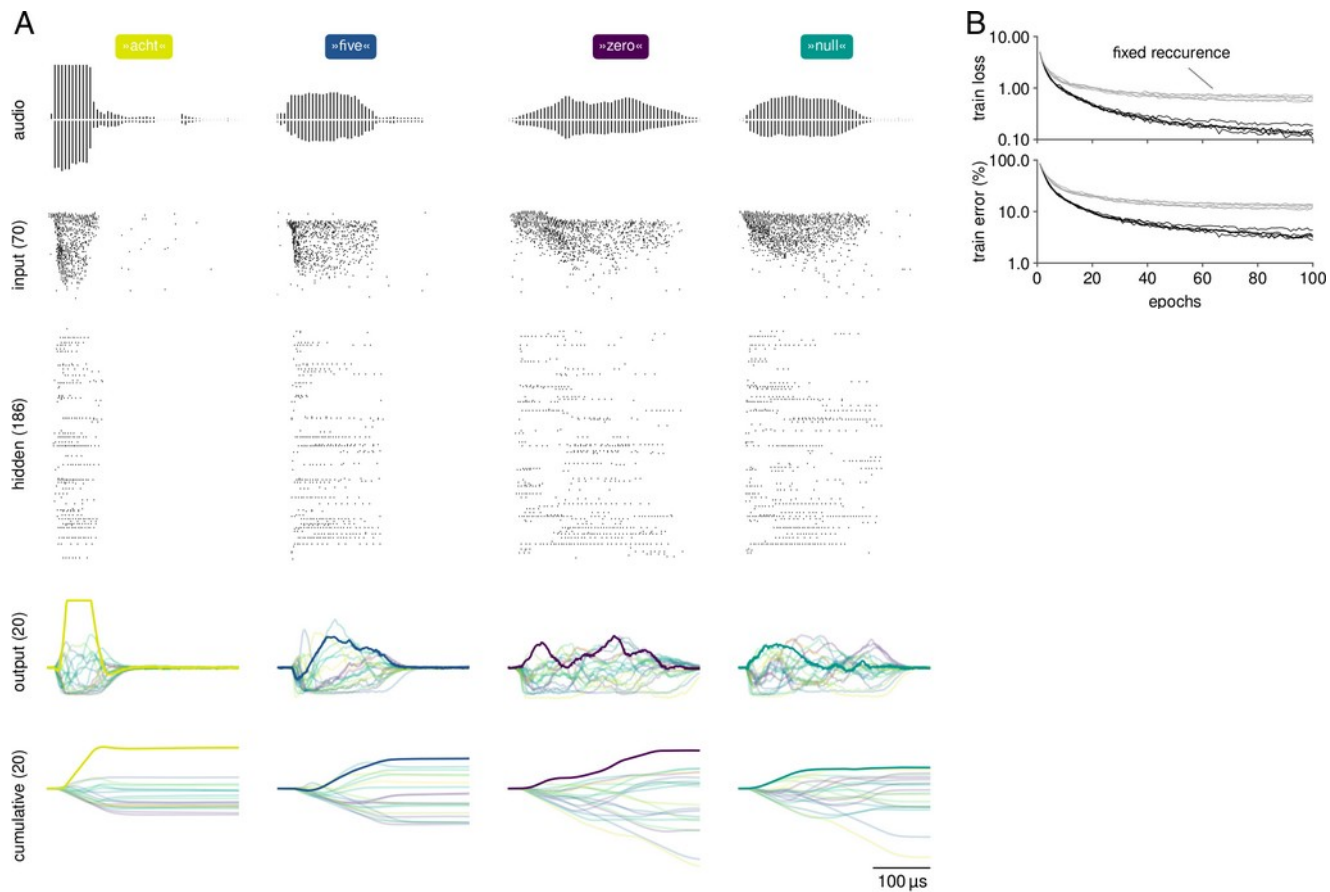
Functional spiking neural networks trained on BrainScaleS-2 analog neuromorphic hardware



Surrogate gradient learning self-calibrates the analog neuromorphic substrate



Speech classification and keyword spotting (SHD)



Cramer, B., Billaudelle, S., Kanya, S., Leibfried, A., Grübl, A., Karasenko, V., Pehle, C., Schreiber, K., Stradmann, Y., Weis, J., Schemmel, J., and Zenke, F. (2022). PNAS

Summary: Voltage aware surrogate gradients can self-calibrate analog neuromorphic substrates



Johannes Schemmel
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Benjamin
Cramer



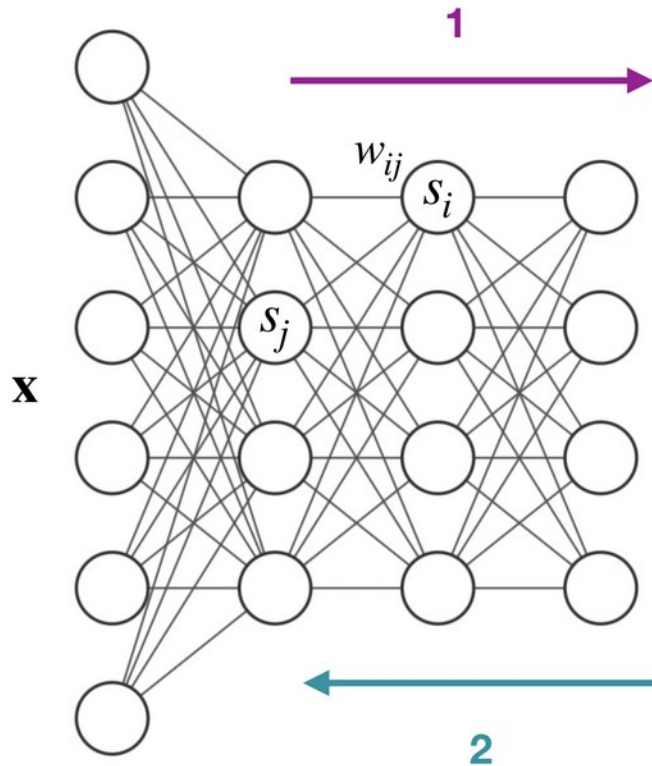
Sebastian
Billaudelle



Cramer, B., Billaudelle, S., Kanya, S., Leibfried, A., Grübl, A., Karasenko, V., Pehle, C., Schreiber, K., Stradmann, Y., Weis, J., Schemmel, J., and Zenke, F. (2022). *PNAS*

Still, training was done offline and used backprop.

Backprop is difficult to implement on neuromorphic systems



Rumelhart et al. *Nature* 1986

1 Nonlinear computation

$$s_i = \sigma \left(\sum_j w_{ij} s_j + b_i \right)$$

\mathcal{L} loss function quantifying good or bad

2 Error Backpropagation (BP)

$$\Delta w_{ij} \propto - \frac{d\mathcal{L}}{dw_{ij}} = - \delta_i \sigma'(s_j)$$

non local

Question

How to train noisy physical networks without backprop?



Axel Laborieux

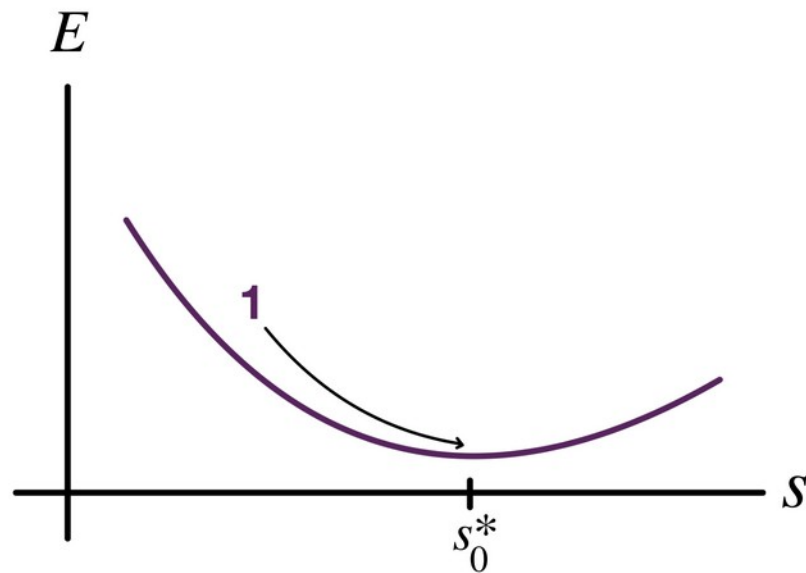
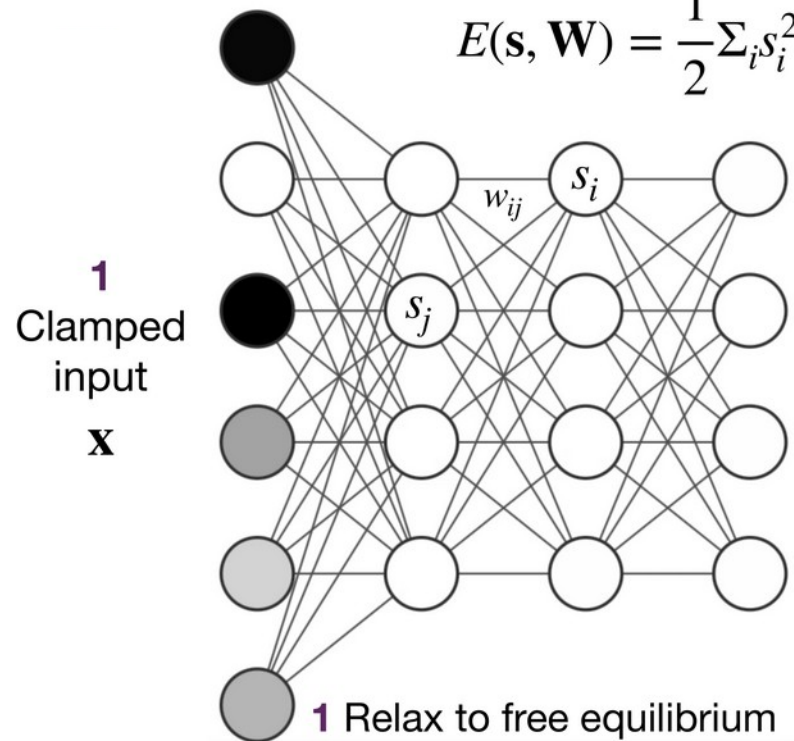
Laborieux and Zenke (2022) *Neurips*

Holomorphic Equilibrium Propagation Computes Exact Gradients Through Finite Size Oscillations

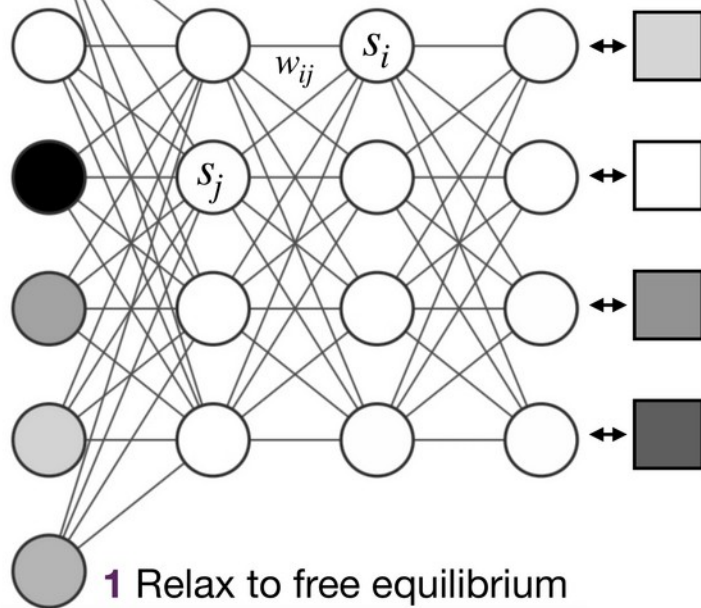


Equilibrium Propagation (EP) is an alternative

$$E(\mathbf{s}, \mathbf{W}) = \frac{1}{2} \sum_i s_i^2 - \sum_{i < j} w_{ij} \sigma(s_i) \sigma(s_j)$$



$$E(\mathbf{s}, \mathbf{W}) = \frac{1}{2} \sum_i s_i^2 - \sum_{i < j} w_{ij} \sigma(s_i) \sigma(s_j)$$

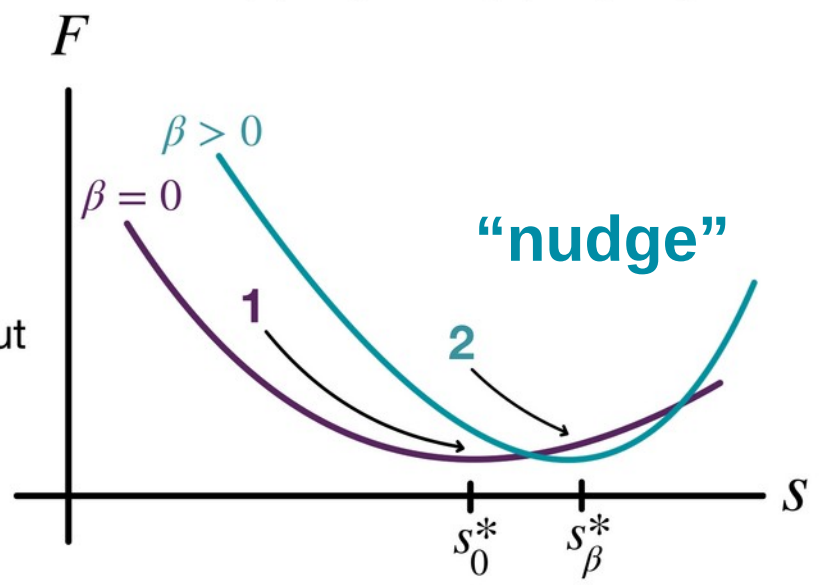


1 & 2
Clamped
input
 \mathbf{x}

1 Free output
2 Nudged output

1 Relax to free equilibrium
2 Relax to nudged equilibrium

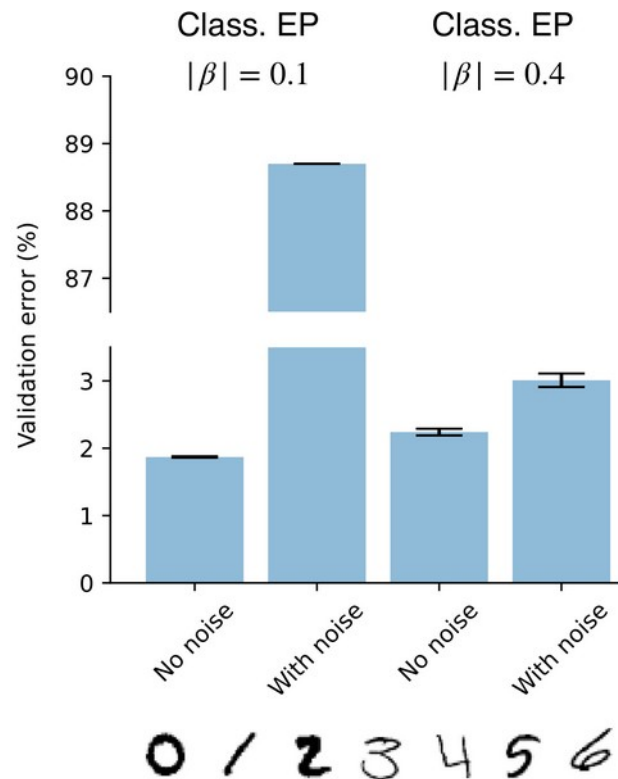
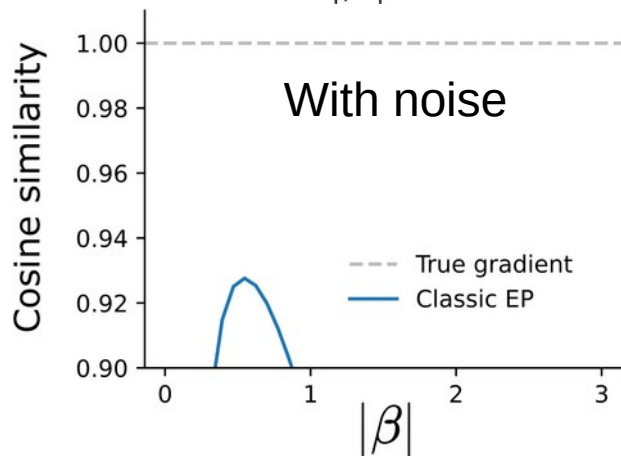
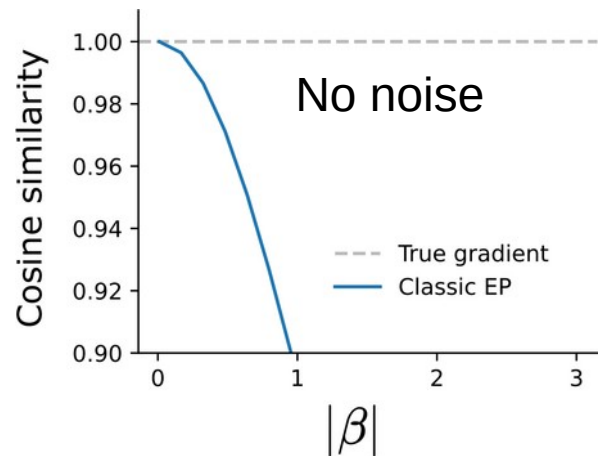
$$F(\mathbf{s}, \mathbf{W}) = E(\mathbf{s}, \mathbf{W}) + \beta \mathcal{L}$$



Local learning rule

$$\Delta w_{ij} \propto \frac{\sigma(s_{\beta,i}^*) \sigma(s_{\beta,j}^*) - \sigma(s_{0,i}^*) \sigma(s_{0,j}^*)}{\beta} \rightarrow - \frac{d\mathcal{L}}{dw_{ij}} \text{ when } \beta \rightarrow 0$$

Classic Equilibrium Propagation is noise sensitive



10

Affiliated with th

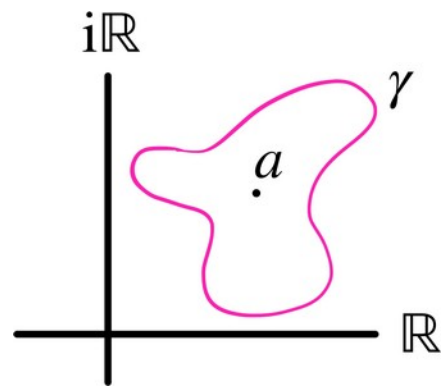
Complex analysis: Derivatives can be expressed as integrals

- Complex differentiability:

$$f'(a) := \lim_{z \rightarrow a} \frac{f(z) - f(a)}{z - a}$$

'holomorphic'

- Cauchy integral: $f'(a) = \frac{1}{2i\pi} \oint_{\gamma} \frac{f(z)}{(z - a)^2} dz$



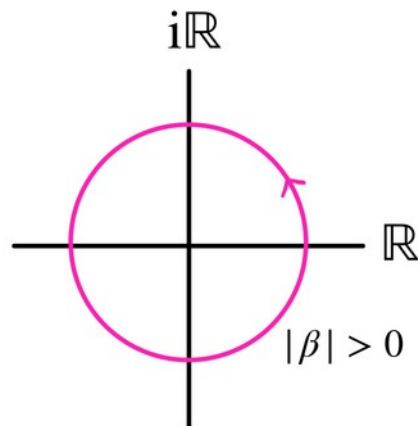
Integration over one oscillation yields local learning rule

Replace derivative by a Cauchy integral

$$-\frac{d\mathcal{L}}{d\mathbf{W}} \Big|_{\mathbf{W}_0} = \frac{d}{d\beta} \Big|_{\beta=0} \left(\sigma(\mathbf{s}_\beta^*) \sigma(\mathbf{s}_\beta^*)^\top \right) \downarrow = \frac{1}{2i\pi} \oint_\gamma \frac{\sigma(\mathbf{s}_\beta^*) \sigma(\mathbf{s}_\beta^*)^\top}{\beta^2} d\beta$$

$\beta \in \mathbb{C}$

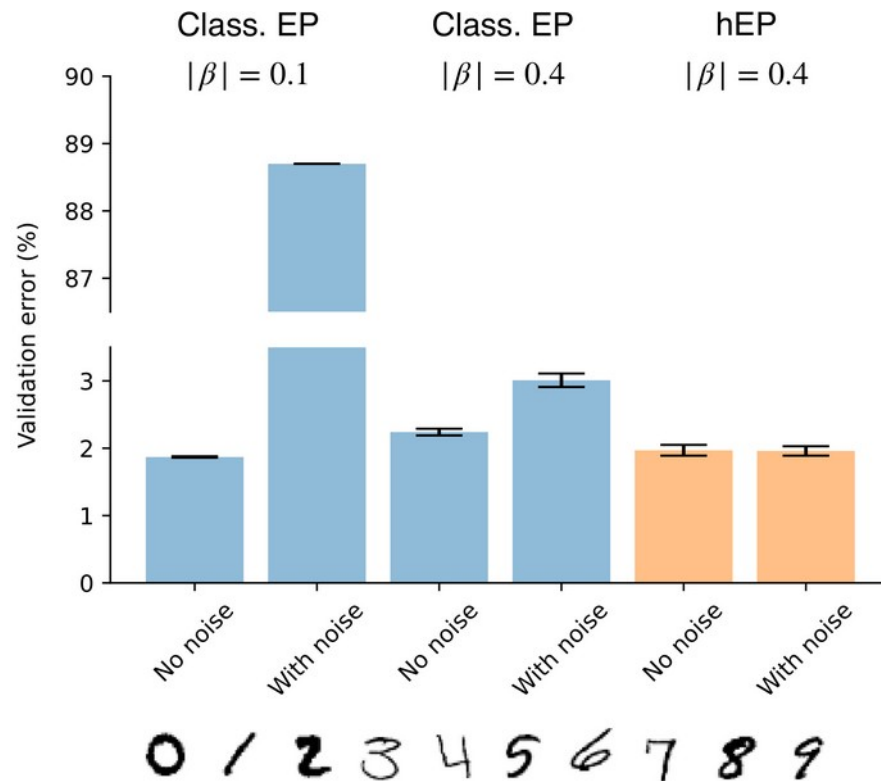
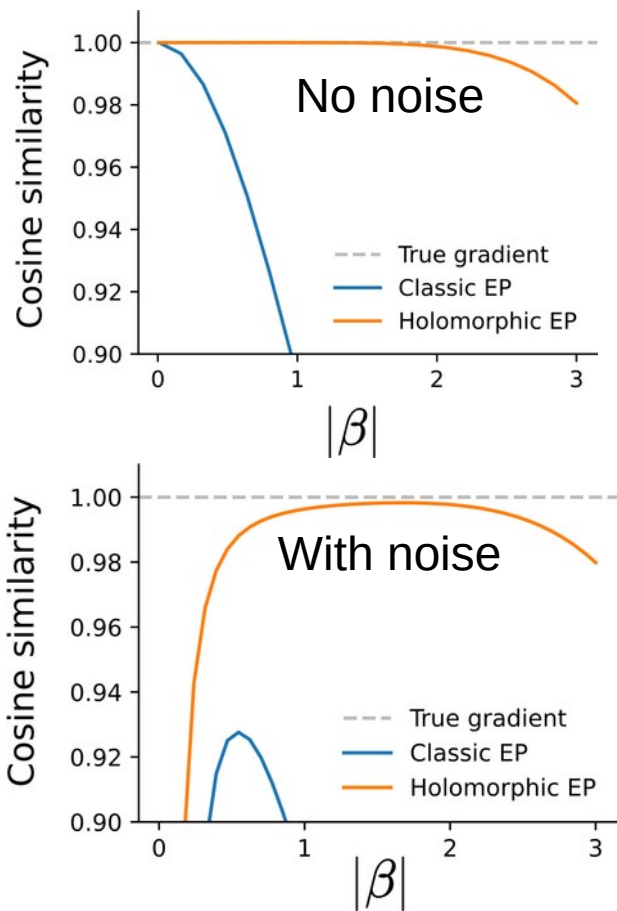
We choose the path: $t \in [0, T_{\text{osc}}] \mapsto \beta(t) = |\beta| e^{2i\pi t/T_{\text{osc}}}$



$$-\frac{d\mathcal{L}}{d\mathbf{W}} \Big|_{\mathbf{W}_0} = \frac{1}{T_{\text{osc}} |\beta|} \int_0^{T_{\text{osc}}} \sigma(\mathbf{s}_{\beta(t)}^*) \sigma(\mathbf{s}_{\beta(t)}^*)^\top e^{-2i\pi t/T_{\text{osc}}} dt$$

Gradient = first Fourier coefficient of nonlinear neural oscillations

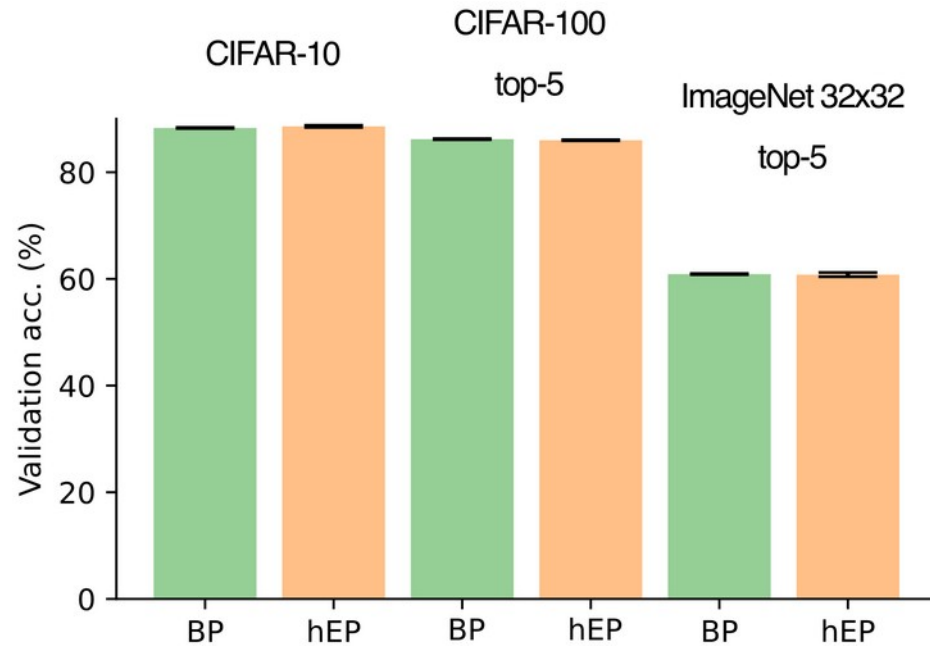
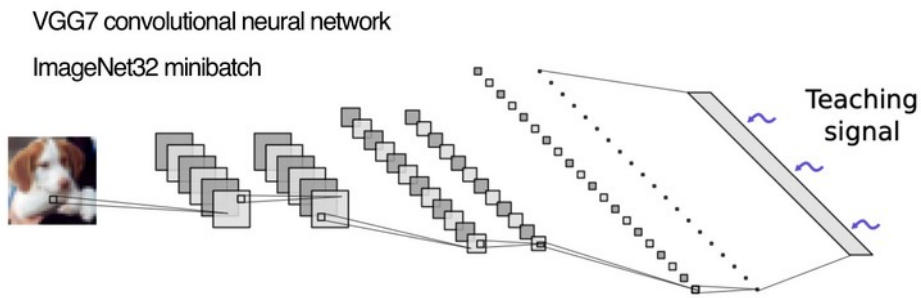
Holomorphic Equilibrium Propagation is robust



10

Affiliated with the Novartis Institutes for BioMedical Research

Holomorphic EP scales to ImageNet (thanks to larger teaching amplitudes)



Summary: Holomorphic equilibrium propagation allows computing exact gradients on noisy physical systems (without backprop)



Axel Laborieux



Laborieux and Zenke (2022) *Neurips*

Holomorphic Equilibrium Propagation Computes Exact Gradients Through Finite Size Oscillations

Follow-up paper dealing with weight asymmetry:

Laborieux and Zenke (2024) *accepted at ICLR*

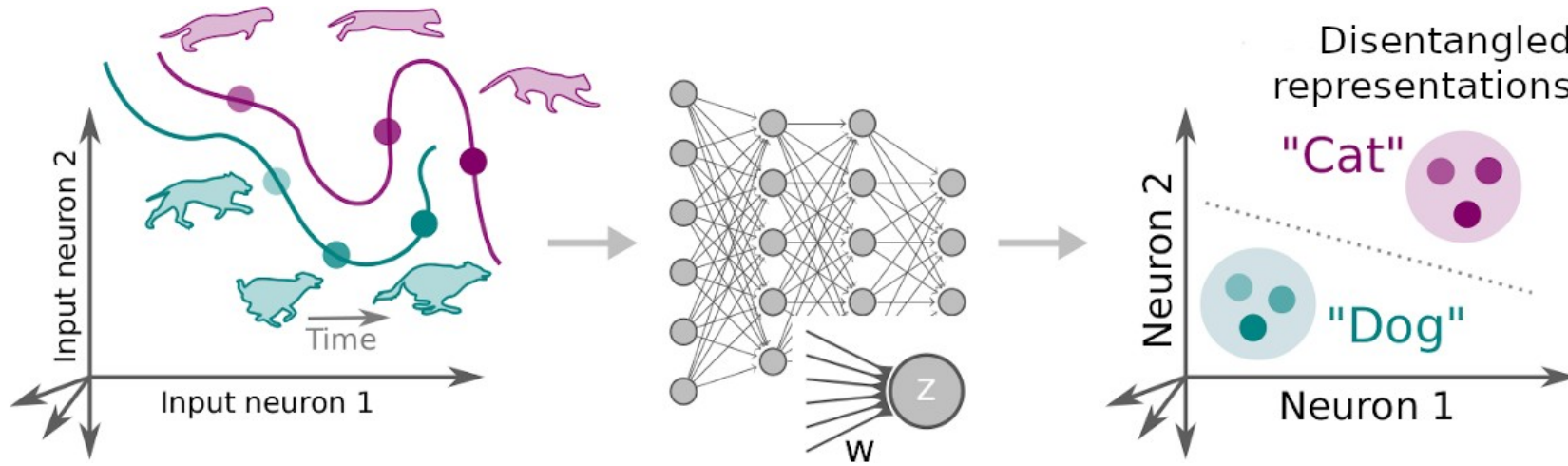
Improving equilibrium propagation without weight symmetry through Jacobian homeostasis



Ongoing work: Make it real

Latent Predictive Learning

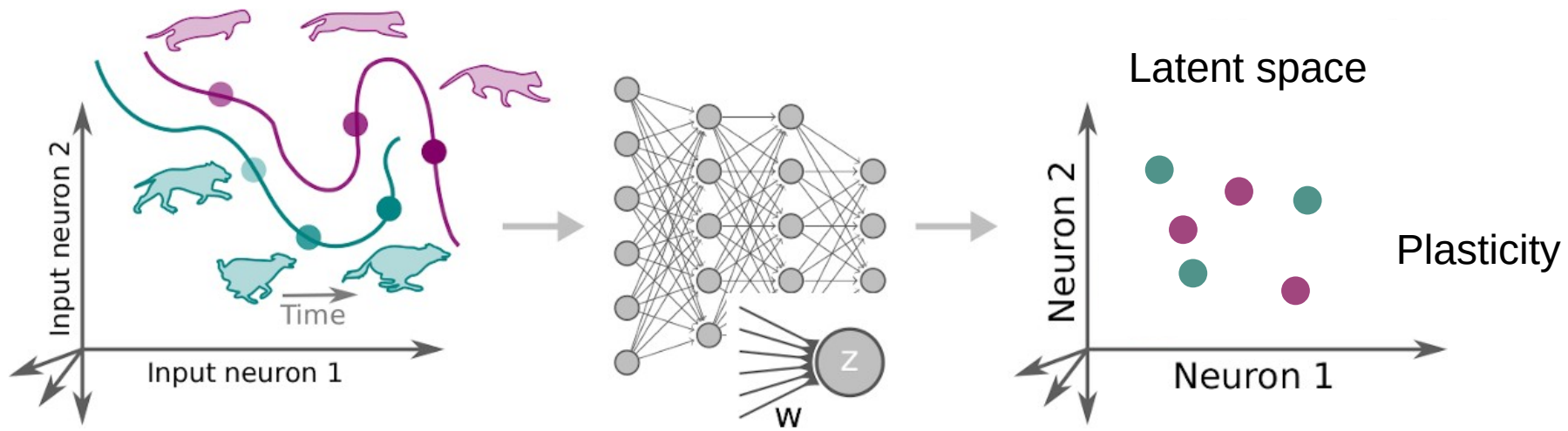
Online self-supervised learning with local rules

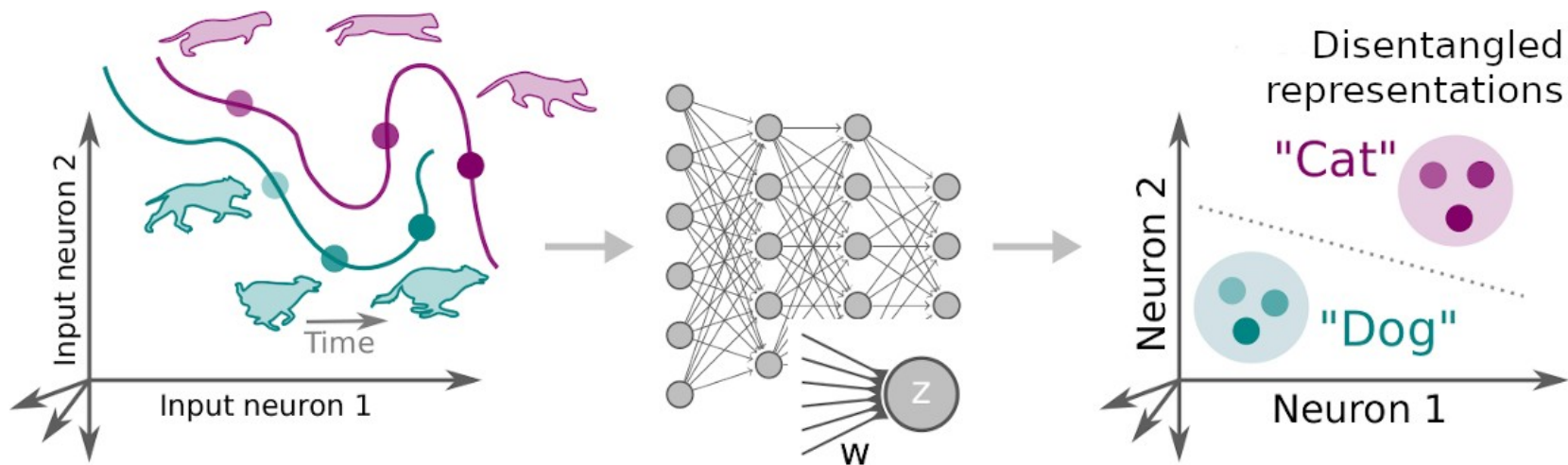


Manu S. Halvagal

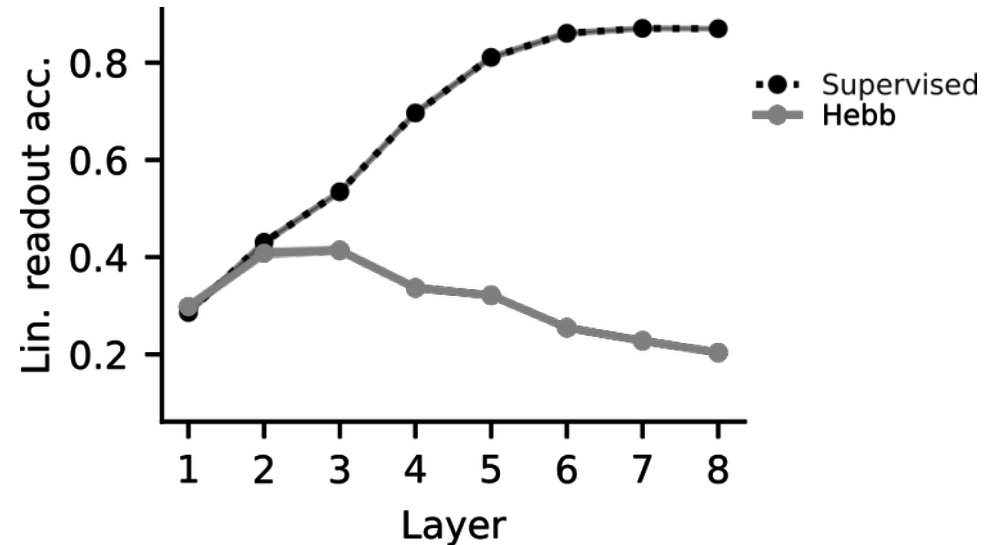
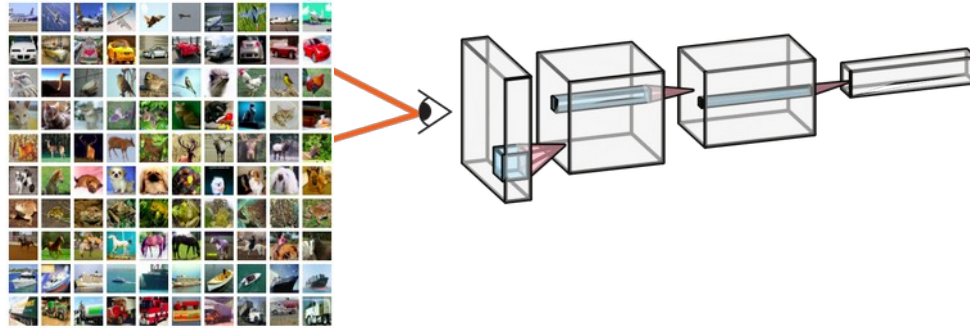
Halvagal and Zenke (2023) *Nat Neurosci*



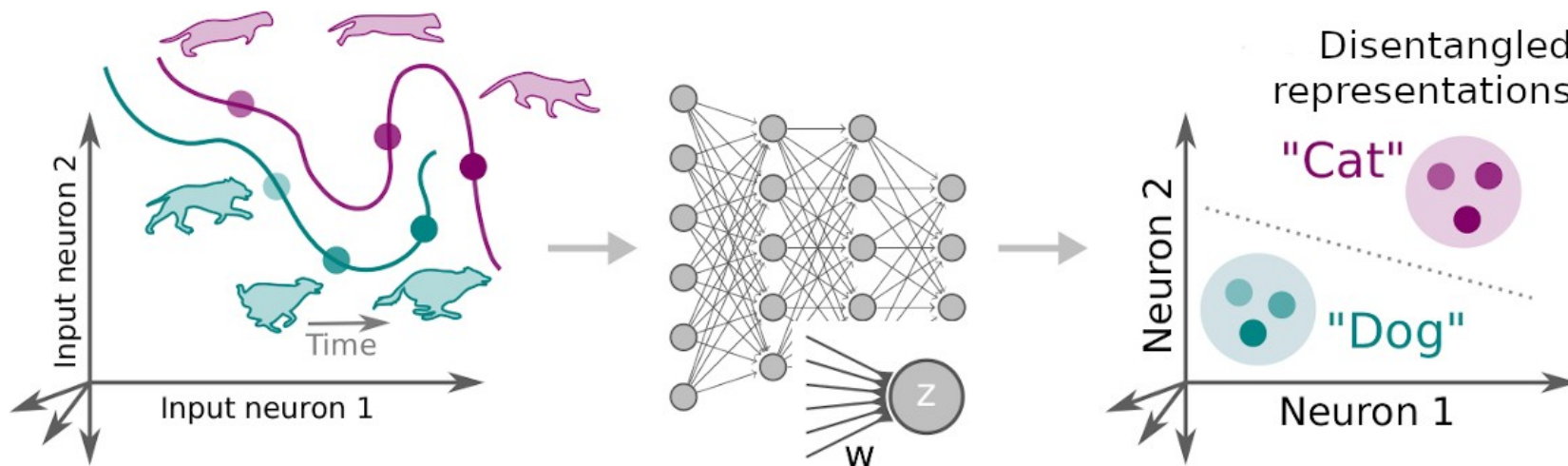




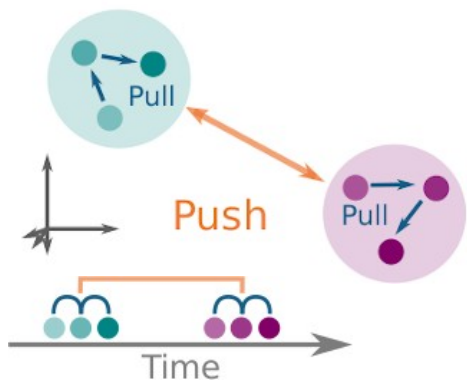
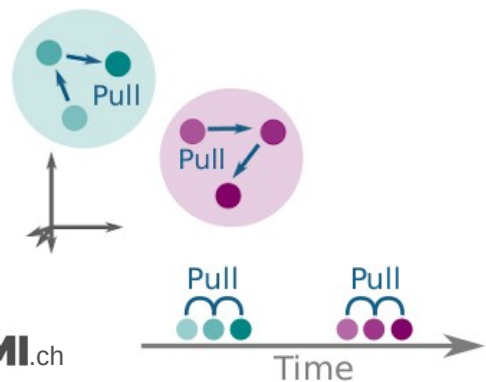
Problem: Hebbian plasticity, a bio-inspired local learning rule, does not learn *good* representations in deep nets



Idea: Optimize for latent space prediction

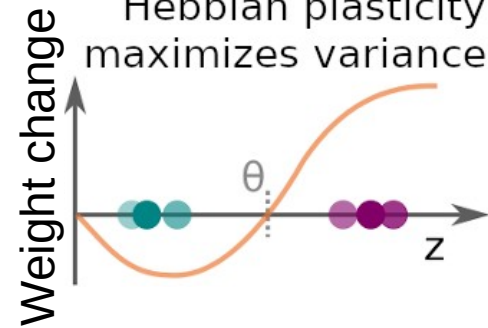


predictive learning



Oja (1982):

Hebbian plasticity maximizes variance



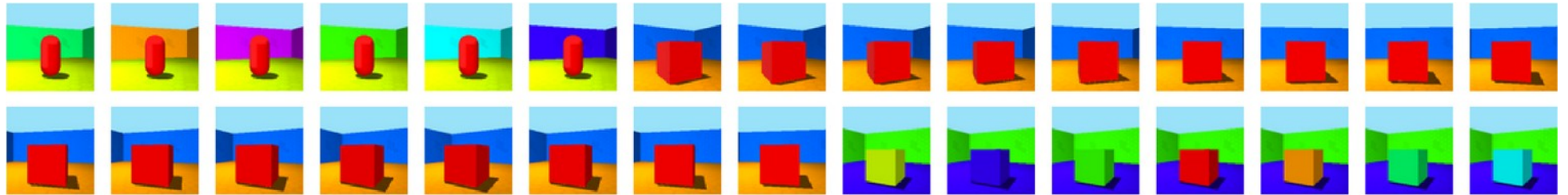
Combining latent space prediction and Hebbian plasticity yields local learning rule

$$\mathcal{L} = \overset{\text{Pull}}{\mathcal{L}_{\text{Pred.}}} + \overset{\text{Push}}{\mathcal{L}_{\text{Hebb}}}$$

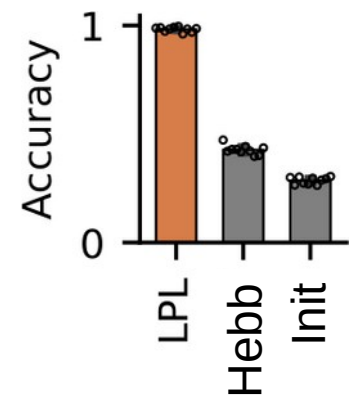
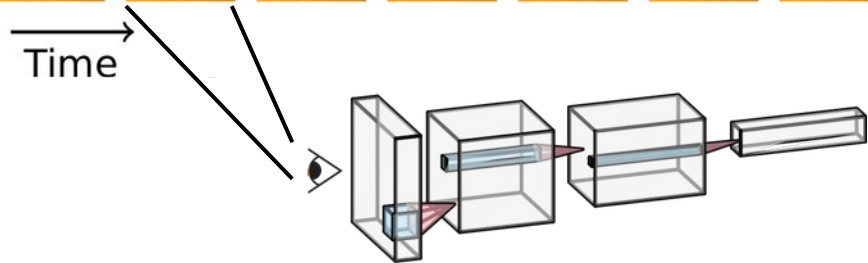
Resulting learning rule is local!

“Latent Predictive Learning LPL”

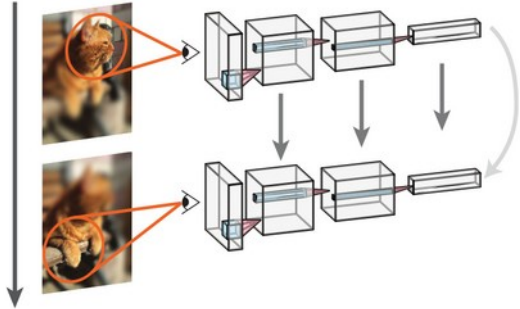
LPL disentangles objects from video data



<https://github.com/deepmind/3d-shapes>



LPL learns invariant representations from augmented images



- VGG-11 model
- Neurons in all layers learn with **LPL**
- No backprop

Original images

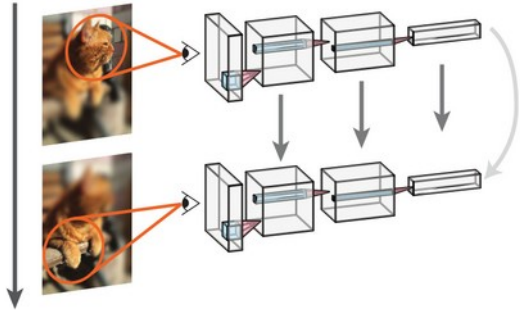


Image sequences



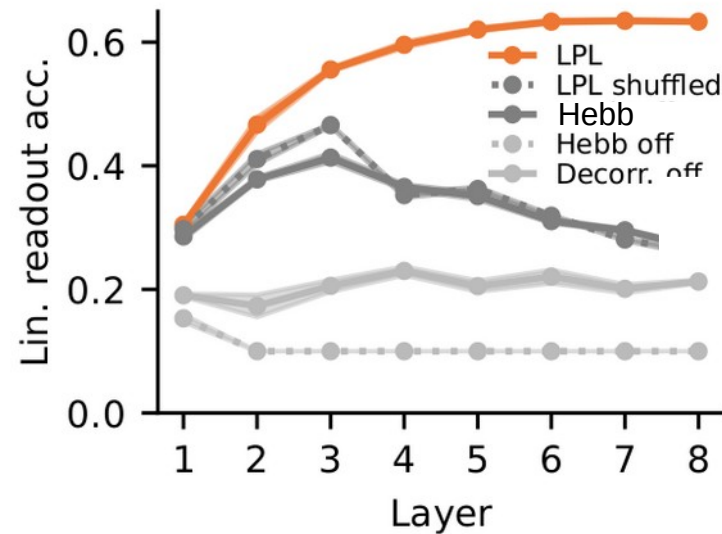
Time →

LPL learns invariant representations from augmented images



- VGG-11 model
- Neurons in all layers learn with **LPL**
- No backprop

After “watching” millions of image sequences ...



LPL can be formulated as a local spiking learning rule

Based on SuperSpike: Zenke & Ganguli (2018)



$$\mathcal{L} = \mathcal{L}_{\text{pred}} + \mathcal{L}_{\text{Hebb}}$$

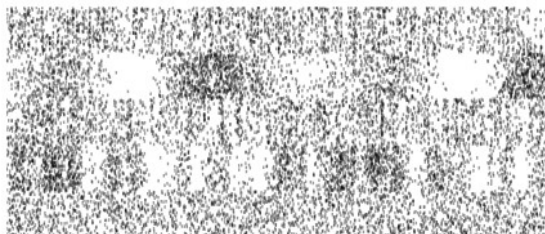
+ inhibitory neurons & plasticity

$$\frac{dw_{ij}}{dt} = \eta \alpha * \left(\underbrace{\epsilon * S_j(t)}_{\text{pre}} \underbrace{f'(U_i(t))}_{\text{post}} \right) \left[\alpha * \left(\underbrace{-(S_i(t) - S_i(t - \Delta t))}_{\text{predictive}} + \underbrace{\frac{\lambda}{\sigma_i^2 + \xi} (S_i(t) - \bar{S}_i(t))}_{\text{Hebb}} \right) \right]$$

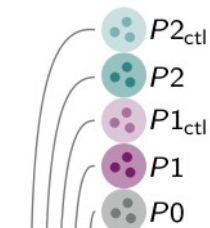
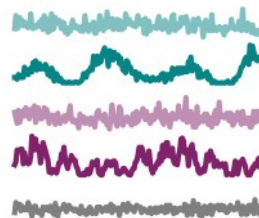
+ η $\underbrace{\delta S_j(t)}_{\text{transmitter-triggered}}$

LPL learns interesting features in streaming data

Input spikes

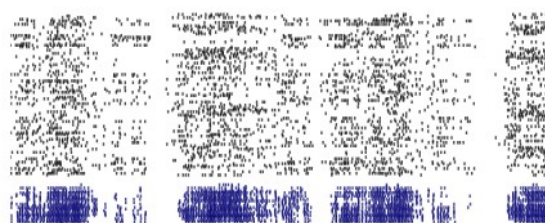


Input signals

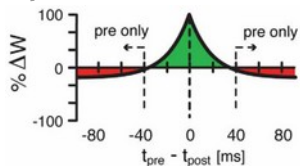
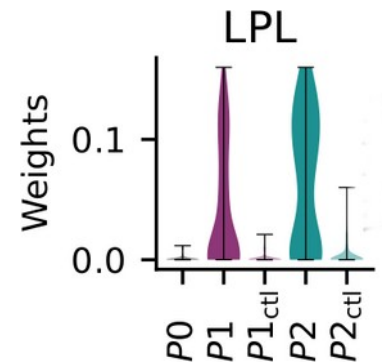
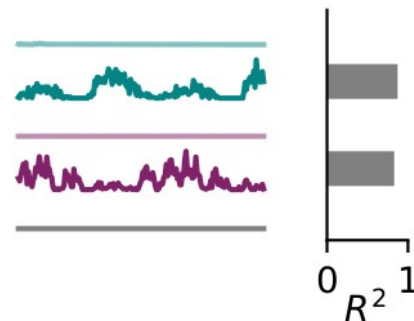


LPL
local learning rule

Network activity



Reconstruction



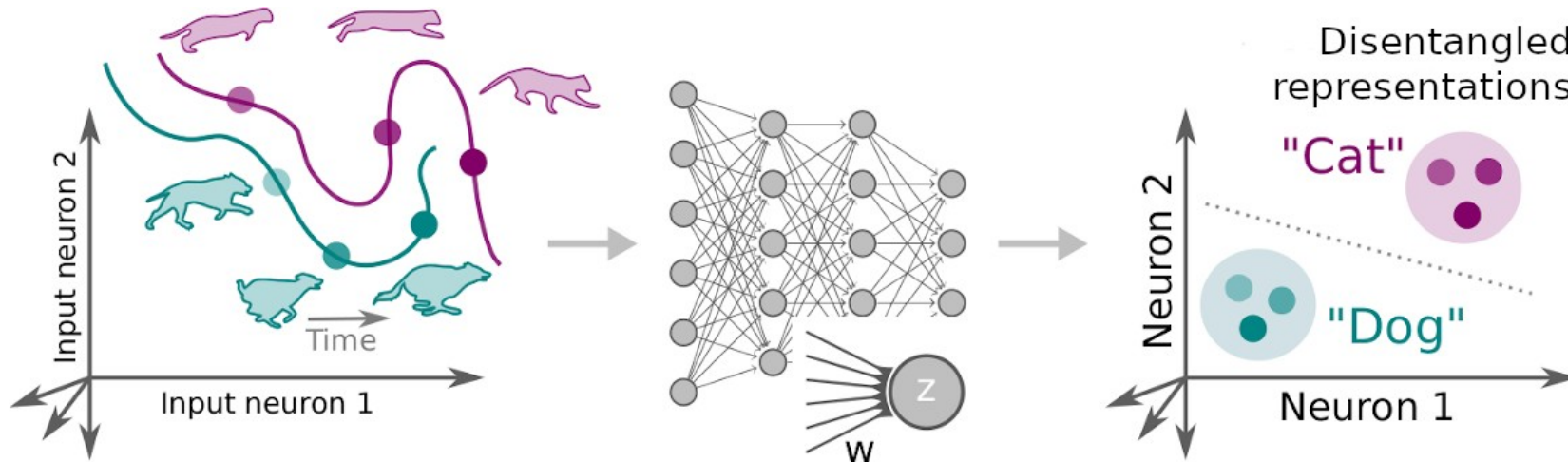
Inhibitory Plasticity

Vogels, T.P., Sprekeler, H., Zenke, F., Clopath, C., and Gerstner, W. (2011)

Latent Predictive Learning: Enables online learning with a local rule without supervision (also works in spiking nets)



Manu S. Halvagal



Halvagal and Zenke (2023) *Nat Neurosci*



Summary

- **Surrogate gradients allow training spiking neural networks end to end**
Neftci, Mostafa, and Zenke (2019) *IEEE SPM*
- **Surrogate gradients can self-calibrate analog neuromorphic substrates and deal with device mismatch**
Cramer, B., Billaudelle, S., Kanya, S., Leibfried, A., Grübl, A., Karasenko, V., Pehle, C., Schreiber, K., Stradmann, Y., Weis, J., Schemmel, J., and Zenke, F. (2022). *PNAS*
- **Holomorphic Equilibrium Propagation allows training noisy analog substrates without Backprop**
Laborieux and Zenke (2022) *Neurips*
- **Latent predictive learning enables online learning without supervision**
Halvagal and Zenke (2023) *Nature Neuroscience*

Way forward for online learning for Edge AI

- Need lightweight online learning rules:
 - Must be robust to noise and heterogeneity
 - No Backprop please!
 - Algorithms like holomorphic EP are promising, but must be practical (real-numbered)
 - Close the gap between self-supervised and supervised learning.
- Need joint efforts in algorithms and hardware development

Thanks!

The team <https://zenkelab.org/team/>



**Axel
Laborieux**



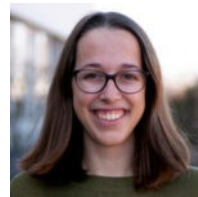
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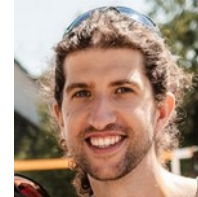
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**Atena
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