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Toward a Formal Semantics for In-Materio Computing Theory

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university of groningen

UEF Ubbo Emmius Foundation

Moving science forward

Professor of Computing in Cognitive Materials

Bernoulli Institute, AI Department *and*
Groningen Cognitive Systems and Materials Center (CogniGron),
University of Groningen

Mission (both of CogniGron and CogniHerbert):

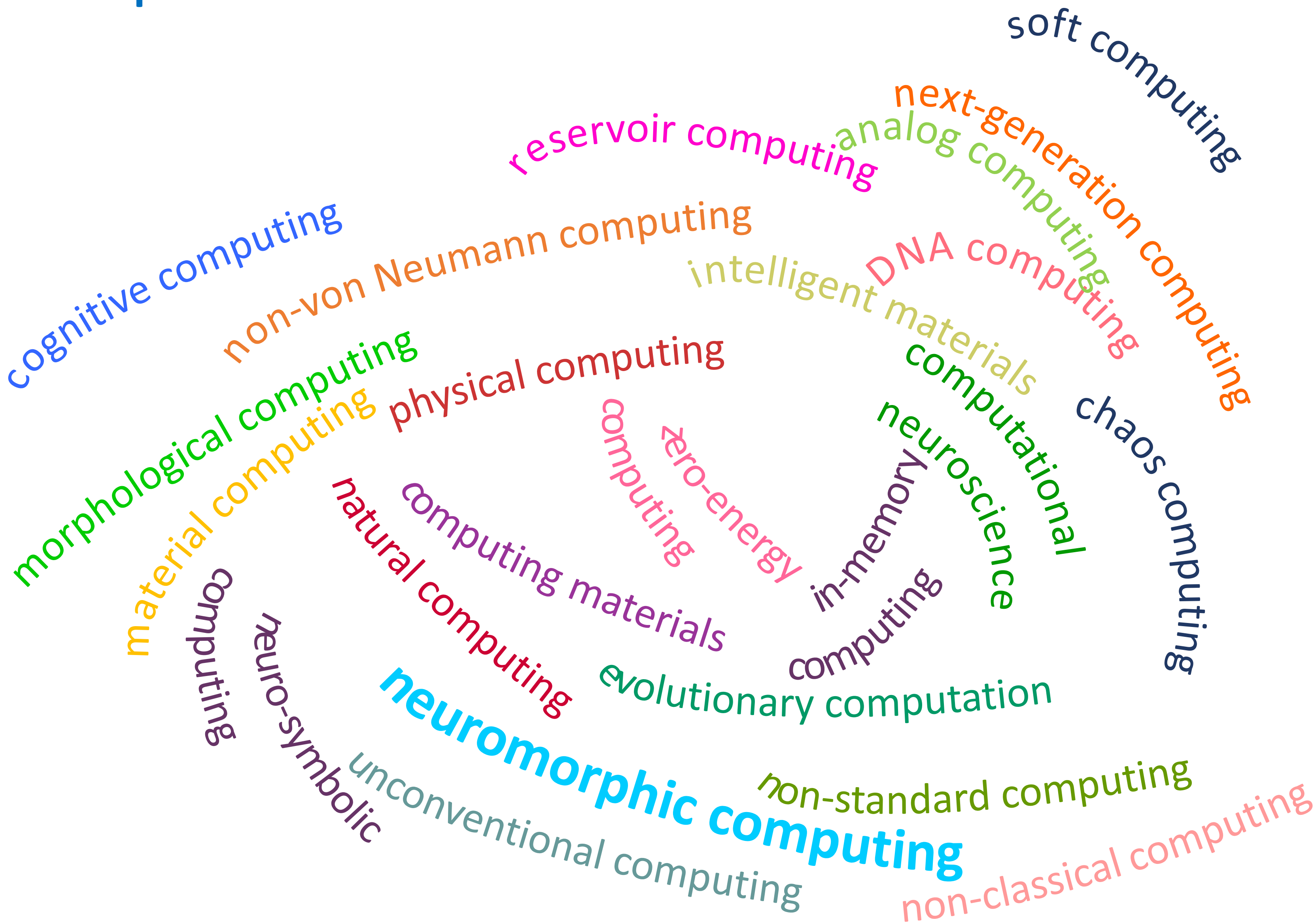
Develop materials-centred systems paradigms for cognitive computing based on modelling and learning at all levels: from materials that can learn to devices, circuits and algorithms

1. Wanted: A General Formal Theory of ‘Computing’

Active lines of 'computing' research

1. Digital computing, including classical quantum computing	from fundamental to applications	"symbolic" computing
2. Energy-efficient deep learning with digital/hybrid spiking neural networks (IBM NorthPole, Intel Loihi) SpiNNaker, BrainScaleS	technology oriented	"neuromorphic" computing
3. Fully analog-electronic spiking neural networks, close to computational neuroscience (DynapsE)	brain-inspired	
4. "compute with whatever nature/physics offers" (materials science, neurobiology, statistical physics, nonlinear systems, philosophy of computation…)	fundamental research	"in-materio" / "physical" / "unconventional" / "natural" /… computing

It’s a whirlpool



Candidate theory proposals (selection)

- Ultrastable systems (Ashby)
- Pattern theory (Grenander, Mumford)
- Free energy agent model (Friston)
- Reservoir computing (Maass, Jaeger)
- Stochastic computing (von Neumann)
- Hyperdimensional computing (Kanerva)
- Neural engineering framework (Eliasmith)
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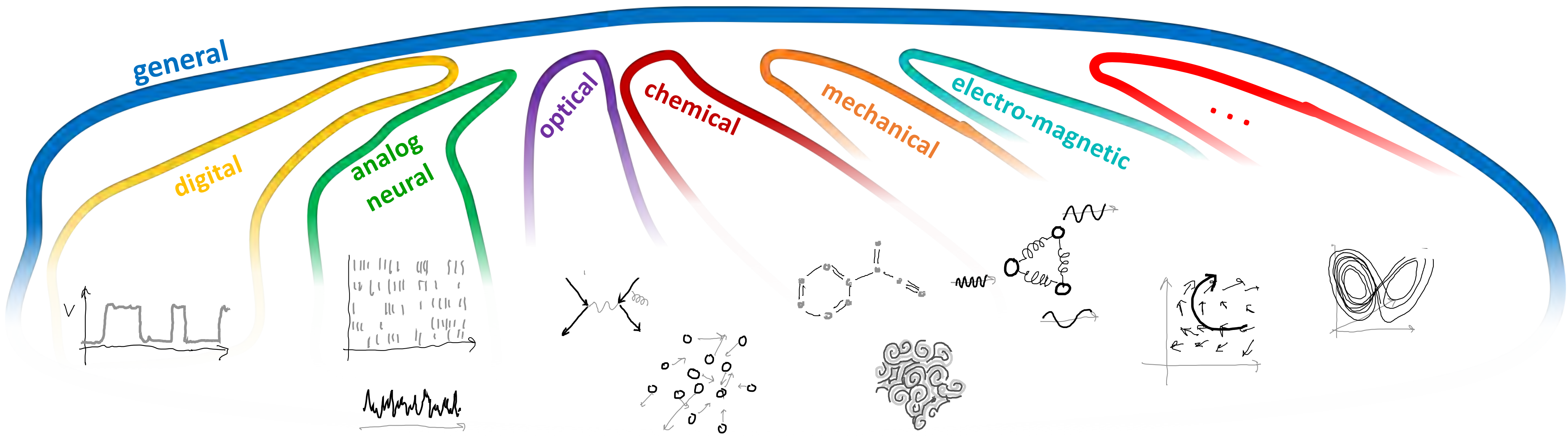
- Wolfram physics (Wolfram)
- Causal sets (Sorkin)
- Commuting diagrams (Horsman, Stepney)
- Neuromorphic compilation hierarchy (Zhang et al)

Too inexpressive
Too abstract / “meta”
Too exclusively ”neuro”
Too informal
Too far away from physics
Too unready
Too symbolic / combinatorial

Jaeger, Noheda, van der Wiel (2023): *Toward a formal theory for computing machines made out of whatever physics offers: long version.* https://arxiv.org/abs/2307.15408

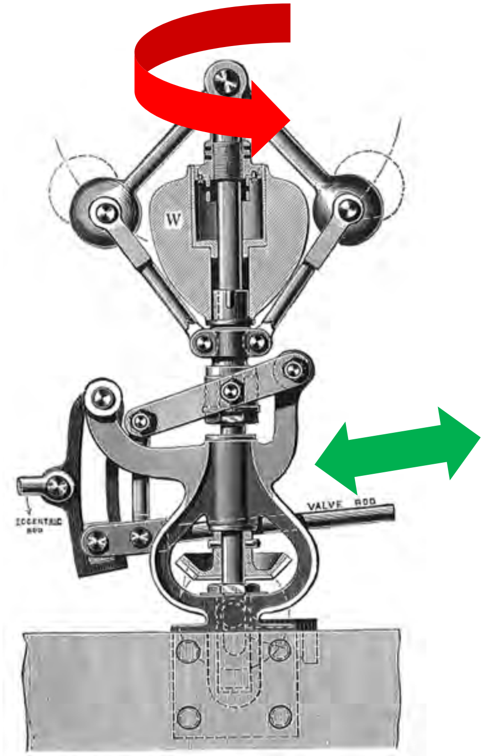
Vision of a general formal theory (GFT) of 'computing' in physical systems

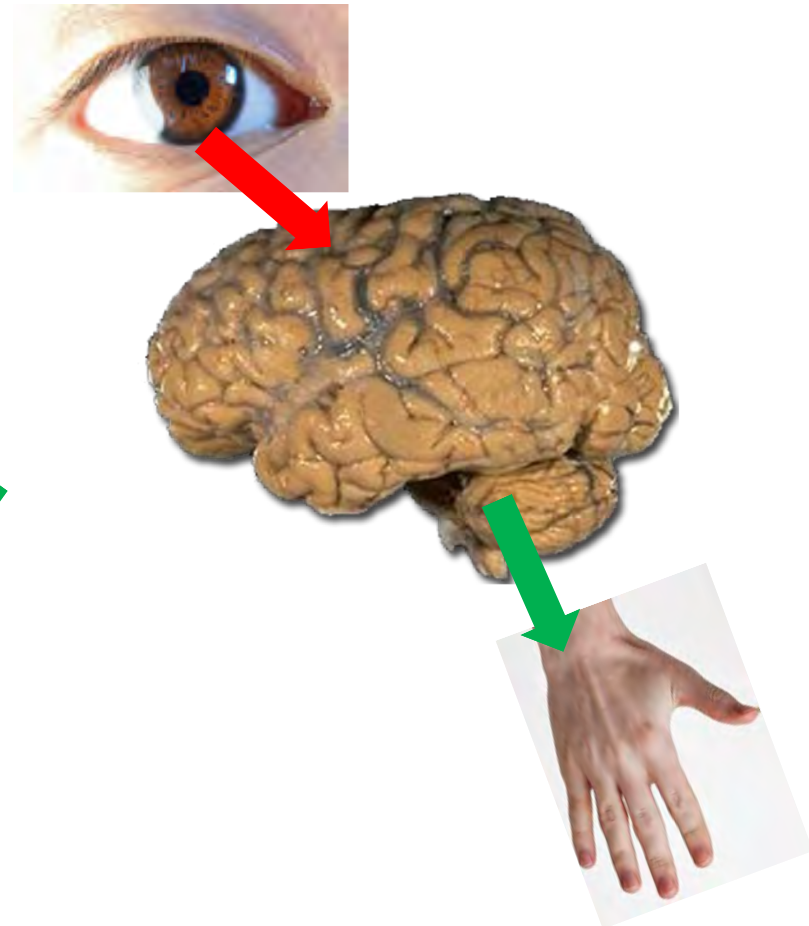


- Unified terminology across disciplines
- Model translations
- Functional invariances and translations in/between physical phenomena
- Simulation models
- System classification
- Digital computing as special case
- Describe natural systems as information-processing

2. Essential components of a GFT

Systems that 'compute'





By (not only my) definition

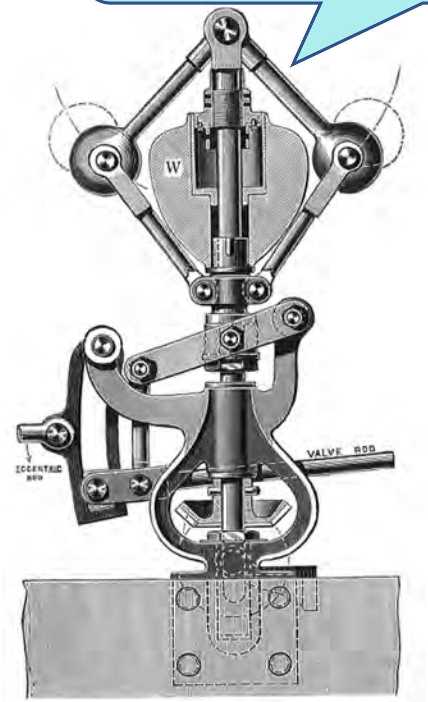
- 'Computing' system must be physical
- 'Computing' system must receive **input** and generate **output**

Image sources collected on last slide

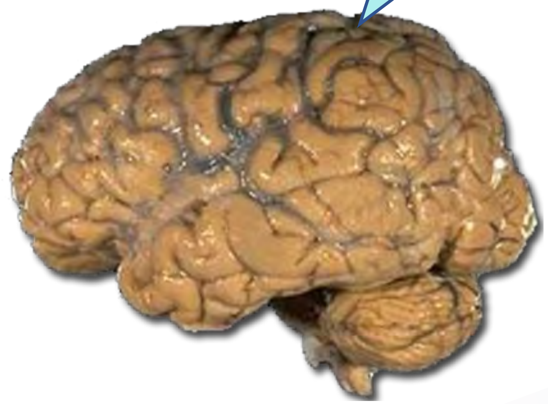
Formal model of ‘computational’ system dynamics

```
k=1;
for n=1 to 5
k = k*n;
end
```

$$F_{cf} = m\omega^2 r$$

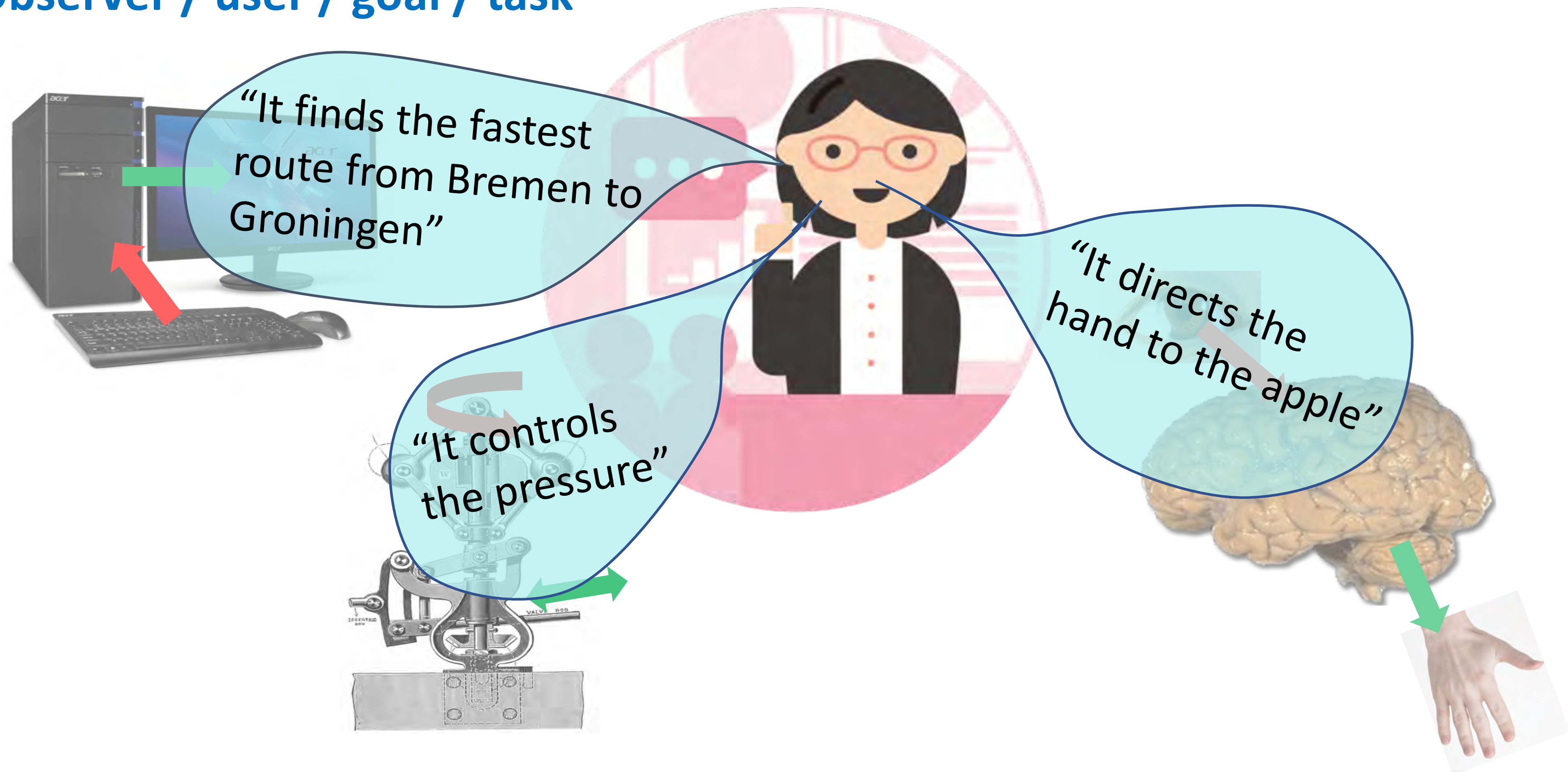


$$\dot{\mathbf{x}} = -c\mathbf{x} + \sigma(\mathbf{W}\mathbf{x} + \mathbf{W}^{in}\mathbf{u})$$



- Formal models needed for engineering design and scientific modeling
- Many mathematical formalisms; many levels of abstraction

Observer / user / goal / task



- 'Computing' implies that the system's input-output is externally *interpreted*
  (Horsman et al., *When does a physical system compute?* Proc. R. Soc. A 470, 2014)
- A 'computation' always serves an externally defined *task*
- Stones or the universe, by themselves, do not 'compute' (no pancomputalism)

The ‘user’…

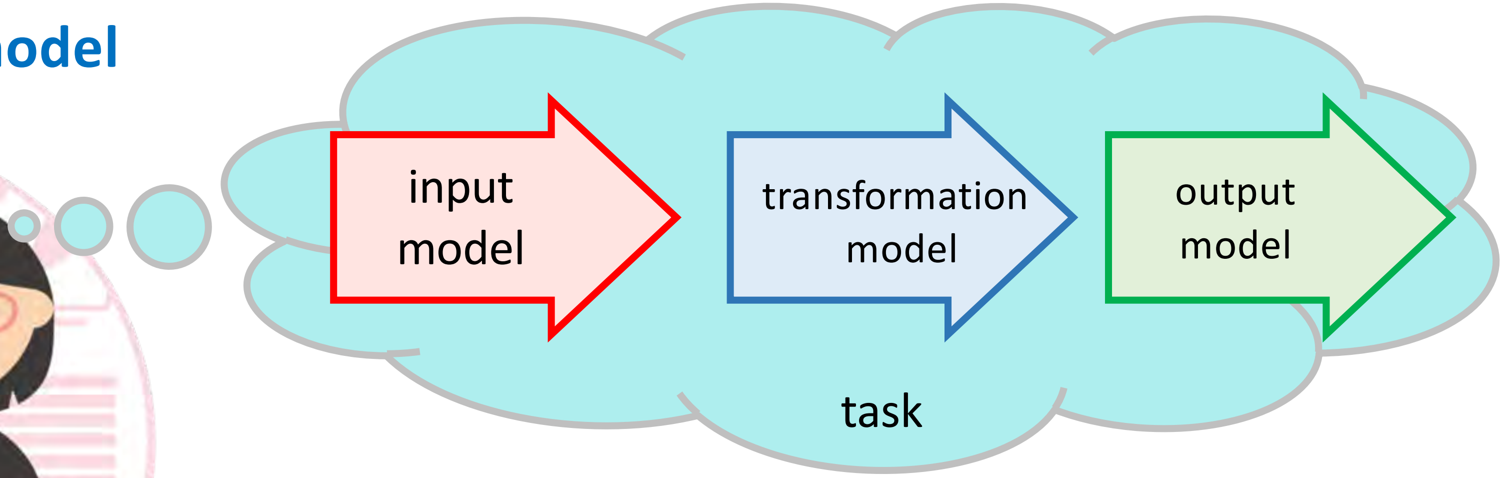
… can be

- a human
- an ‘intelligent’ robot
- an ‘intelligent’ software agent
- a community of users
- another module in a multi-component ‘computing’ system
- …
- general abstraction: computational entity (Horsman et al 2014)

Detail and discussion:

Jaeger, Noheda, van der Wiel (2023): *Toward a formal theory for computing machines made out of whatever physics offers: long version.* https://arxiv.org/abs/2307.15408

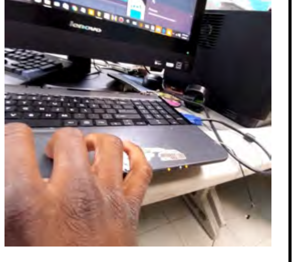
Computational model



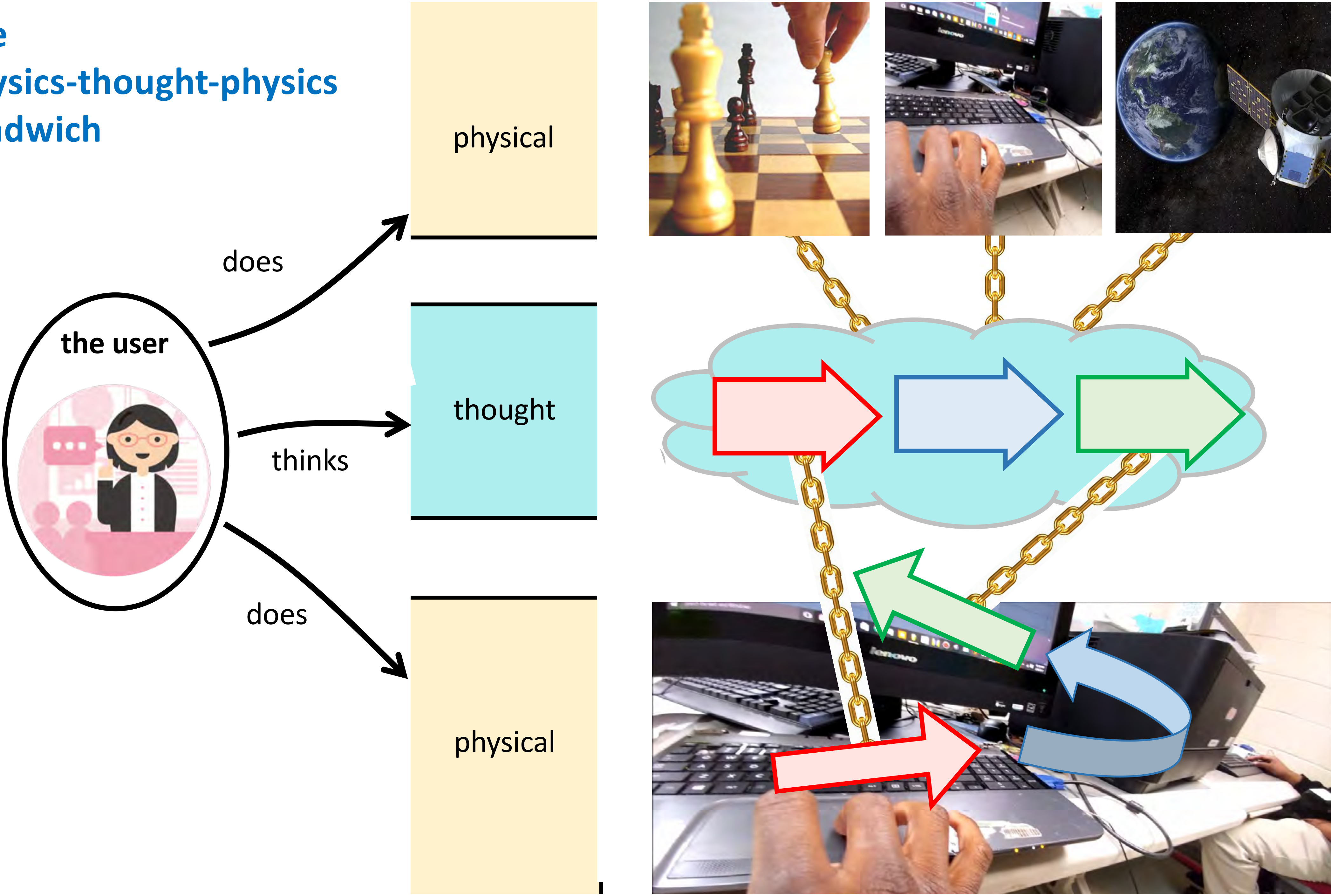
The user commands on a computational model of a task, comprising

- an input model (like thinking of Bremen, car, Groningen)
- an output model (like thinking of a list of road legs)
- a transformation model, explicit or implicit, can take many forms, e.g.
  - a function or relation between inputs and outputs,
  - an optimization criterion,
  - just *anything* that allows assessing goodness of output given input: whether / how task is achieved

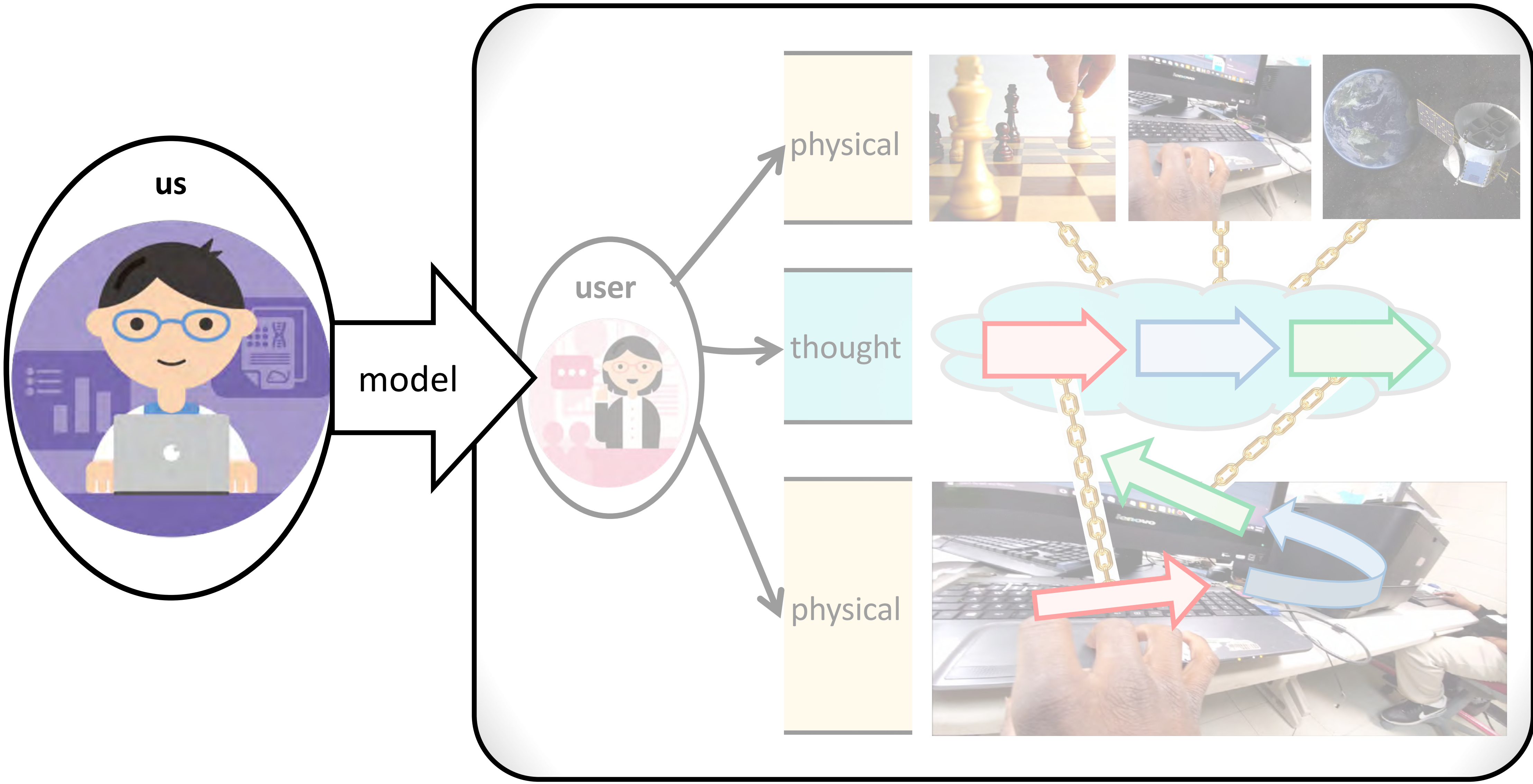
Many types of tasks (only three shown)

task type	**input (mental / model)**	**transformation (mental / model)**	**output (mental / model)**	**task execution (real-world)**
planning	desired goal conditions	simulation of candidate actions	model of pro-posed action or action sequence (‘plan’)	decide for one proposed plan and carry it out 
info query	question, prompt, feature key	search in available knowledge	data matching the query key	enter input, press start, read output (the physical computing *is* the real-world task execution) 
online control	sensor signals & reference signal	apply a ‘filter’	control signal	controlling a physical ‘plant’ 

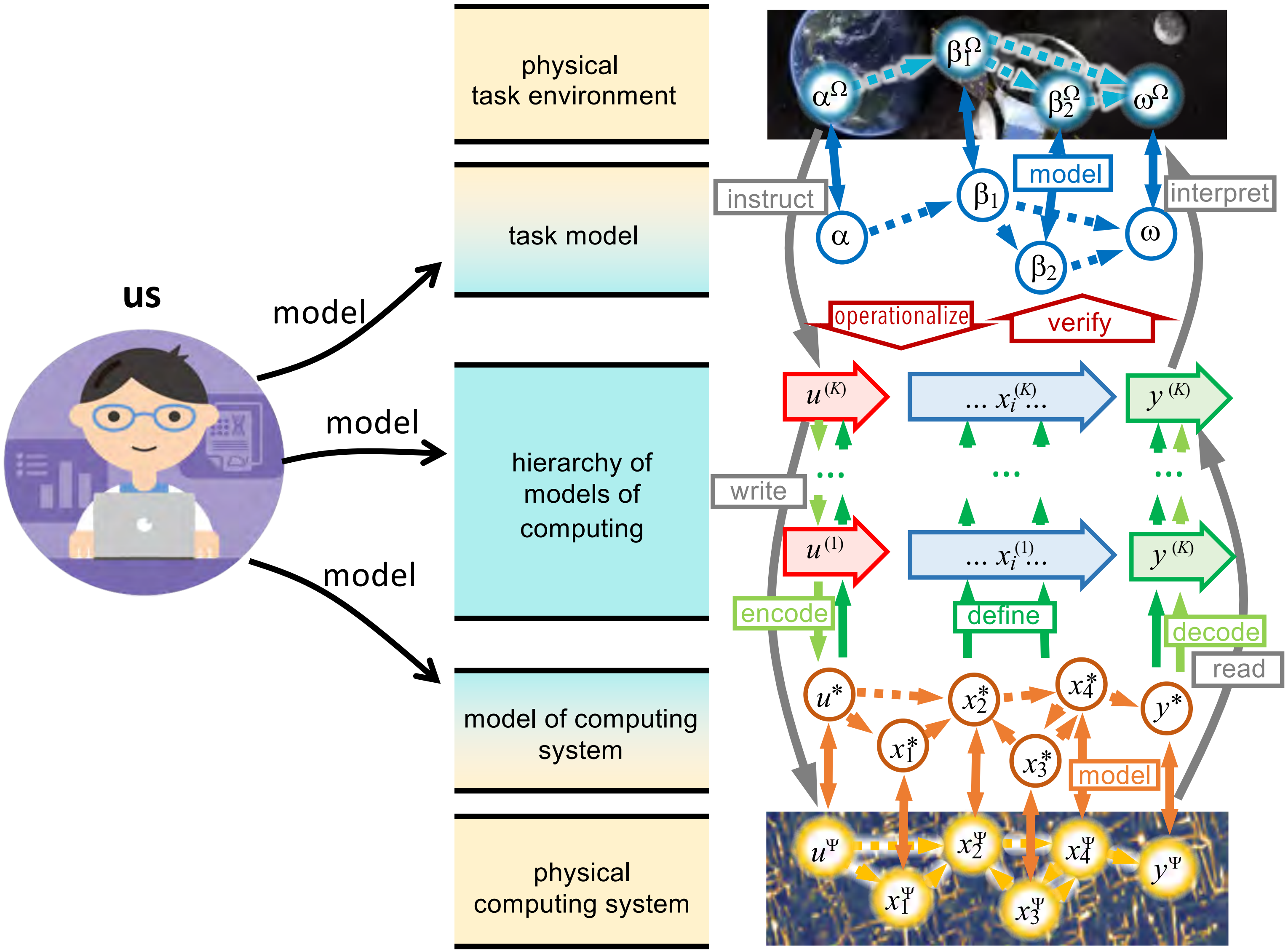
The physics-thought-physics sandwich



Our theory-building task



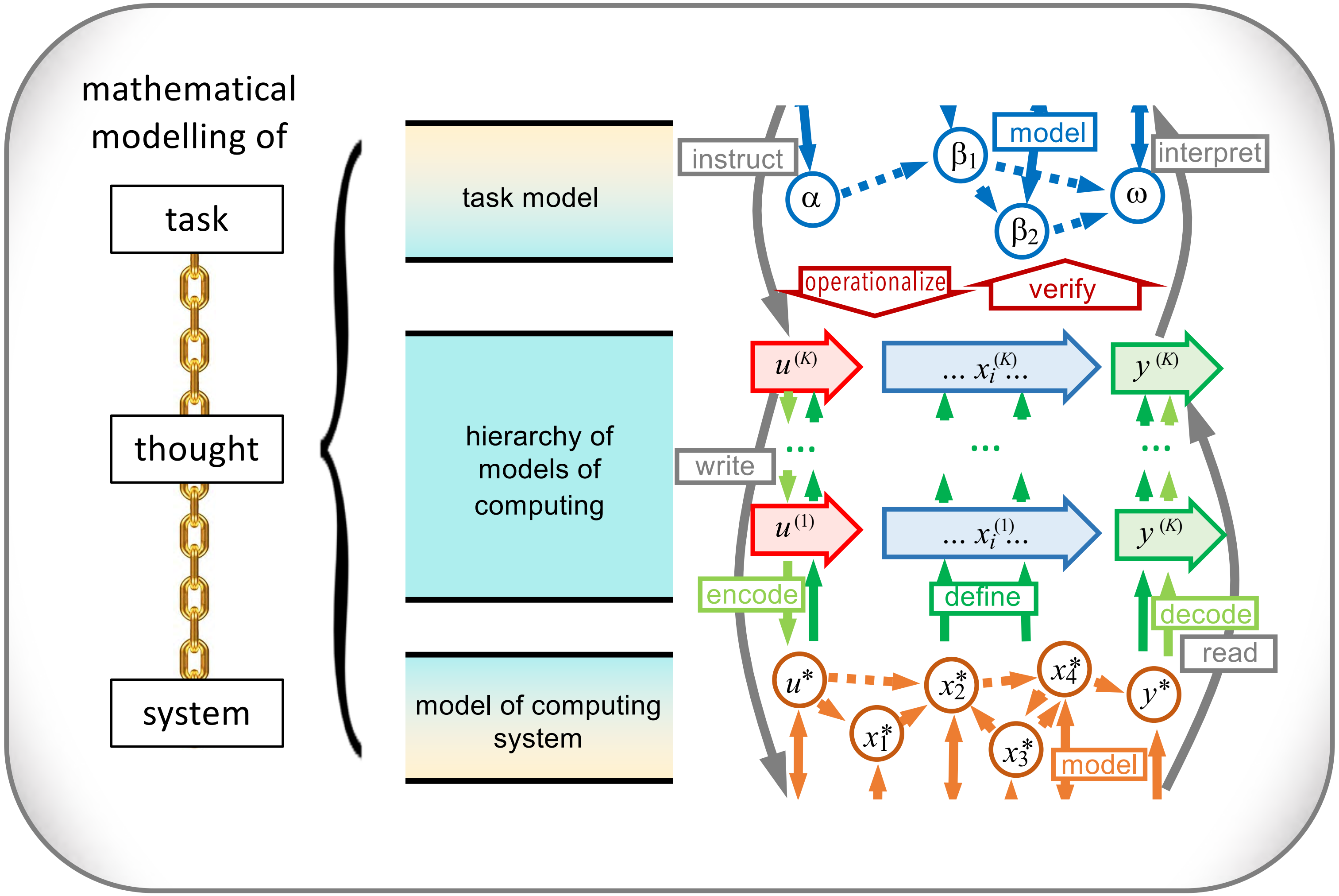
A complex situation



Jaeger, Noheda, van der Wiel (2023)
arxiv.org/abs/2307.15408

A complex to-do

essential
components
of a
GFT

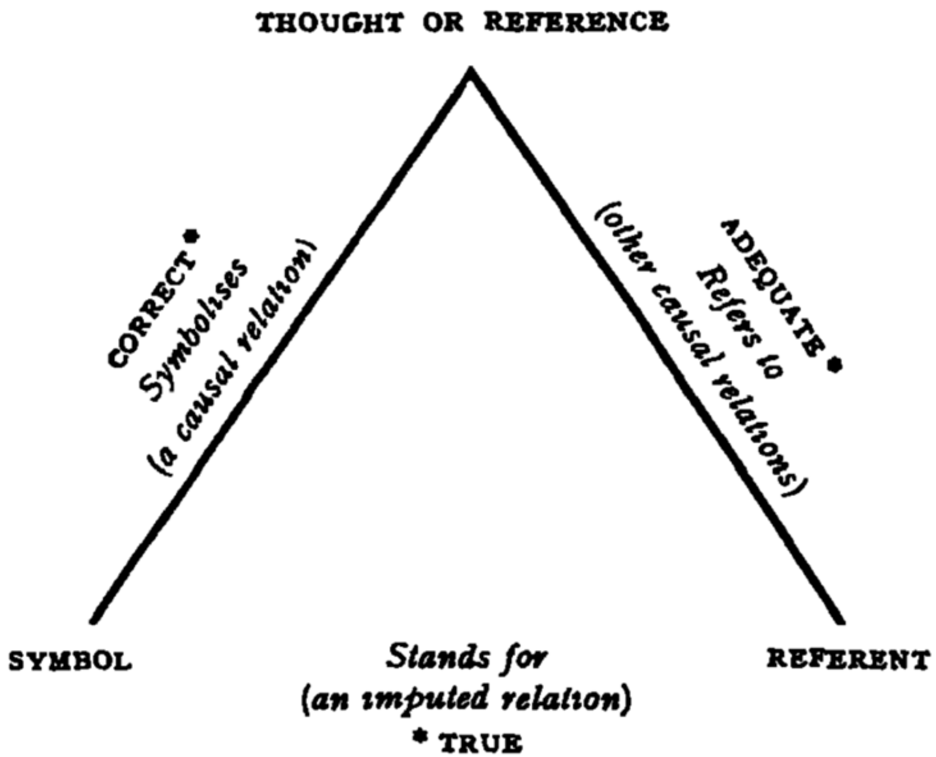


3. A primer on the semantics of ‘semantics’

Semiotics

A field of its own standing

- Goal: understand how 'signals', 'signs', 'symbols' acquire 'meaning' … in real-world life
- Examples of signs: Words, emblematic pictures, memes, traffic signs, gestures
- Hosted in philosophy, linguistics, psychology, social sciences, architecture and arts
- The core meme of semiotics: the semiotic triangle
- The classic classic: C. K. Ogden & I. A. Richards, *The meaning of meaning: A study of the influence of thought and of the science of symbolism*. Harcourt, Brace & World 1923
- Modern classic: U. Eco, *A theory of semiotics*. Indiana University Press 1979 (transl. from Italian original 1968)
- Not mathematically formalized
- Not for us on this occasion

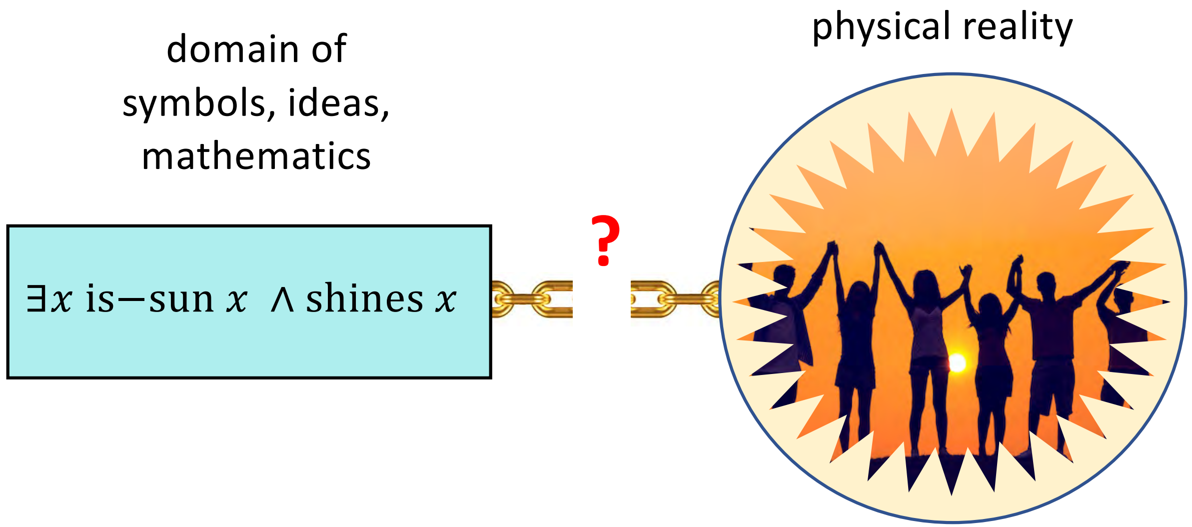


Ogden & Richards 1923

A catch

- A *scientific* theory of semantics needs a *formal mathematical* account of the symbol-referent relation

- Ontological mismatch:
  - symbols belong to the domain of ideas (Turing says *mind*)
  - real-world ‘referents’ are… real-world, … not ideas
  - symbols cannot *formally* connect with physical things



The escape trick

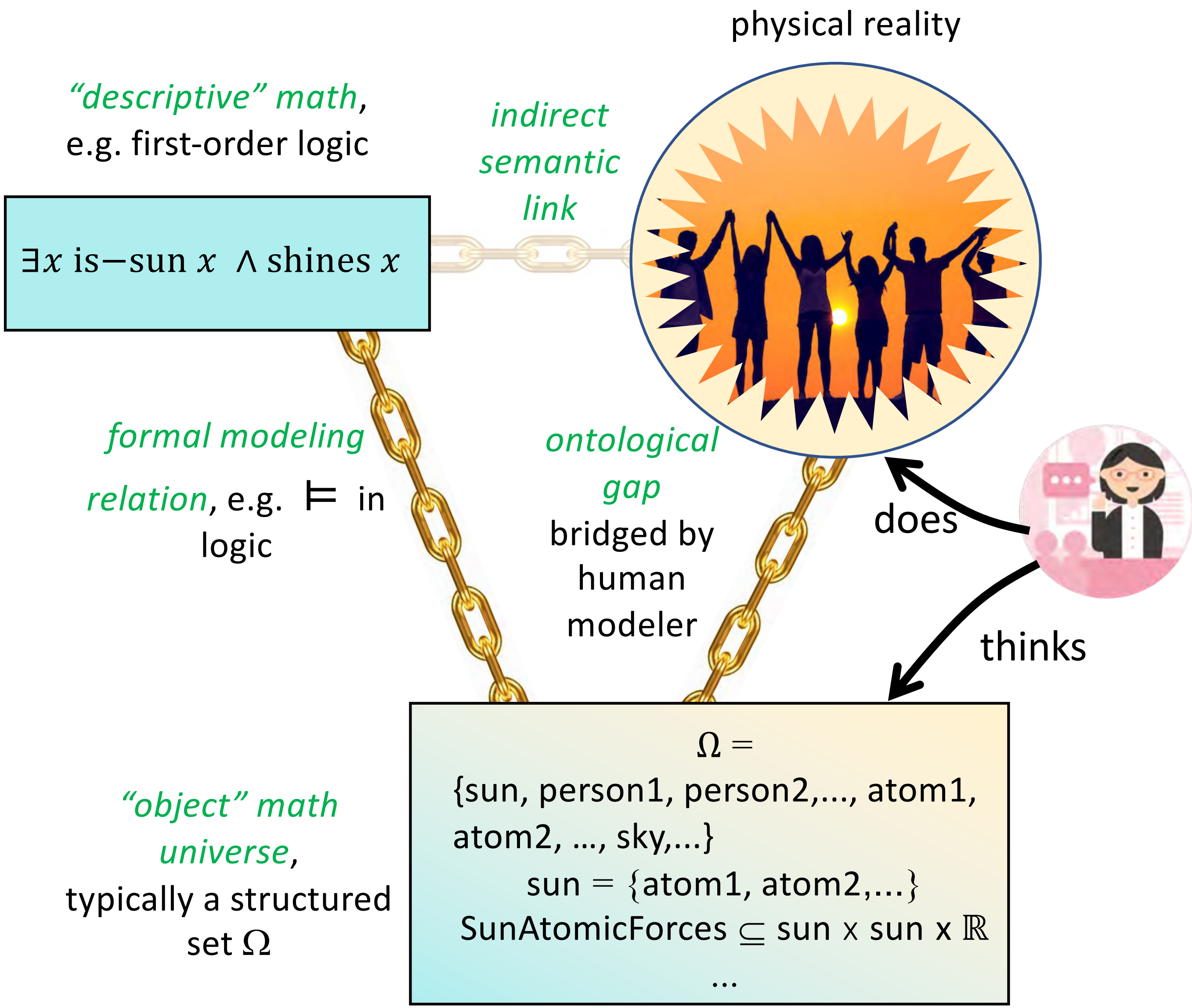
A formally defined semantic relation is possible! ...

...between

- a 'descriptive' mathematical language

  and

- a mathematically defined surrogate "world of referent objects"



Existing pairs of descriptive – object math formalisms

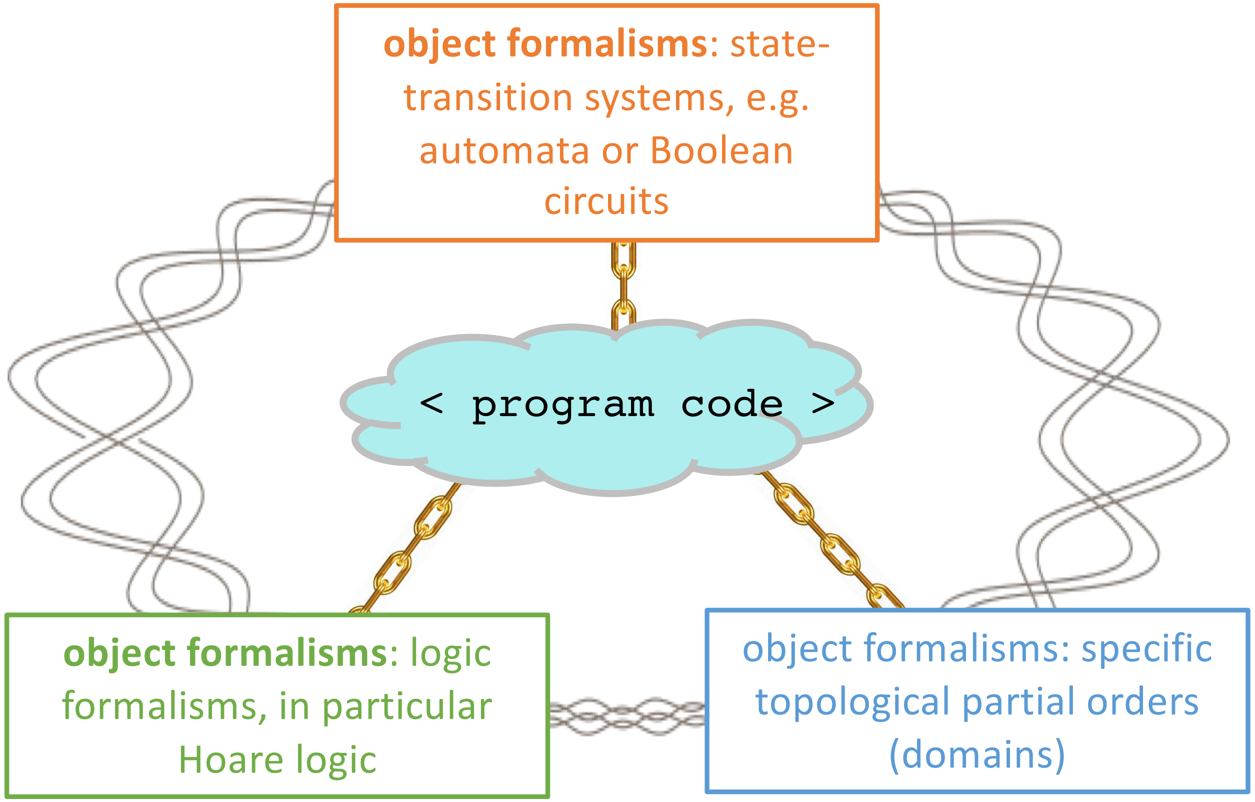
general schema	classical math, comp. science	probability theory	computer science	dynamical systems	GFT
‘descriptive’ math formalism	logic formalism, e.g. first-order logic or a modal logic	probability distributions	programming languages, Turing machines, λ-calculus	qualitative topological phenomena: attractors, bifurcations, …	?
formal modeling relation	$\models$ the model relation in formal logics	random variables	mapping procedures to cont. f’s on domains	ongoing research on global dynamics, e.g. Morse-Smale theory	?
corresponding mathematical ‘object universe’	very richly structured sets, ‘S-structures’ or ‘possible worlds’	probability spaces $(\Omega, \mathfrak{B}, P)$	domains (richly structured partially ordered sets with topology)	fine-grained dynamics specs, like ODEs, PDEs, iterated maps, stoch. diff. eqns	?

H. Jaeger, *Toward a Generalized Theory Comprising Digital, Neuromorphic, and Unconventional Computing*. Neuromorphic Computing and Engineering 1(1), 2021

Adding complexity / confusion: other ways to think about semantics

Computer scientists have 3 ways and 3 motivations to define what program code 'means'

1. Operational semantics:
   - matching code execution against a machine model
   - purpose: checking practical feasibility and correctness
2. Logical semantics:
   - matching execution steps against logical pre/post conditions
   - purpose: verify that a program satisfies a task
3. Denotational semantics:
   - pin down the essential mathematical 'meaning' of symbolic information processing procedures
   - purpose: classify, compare programming languages

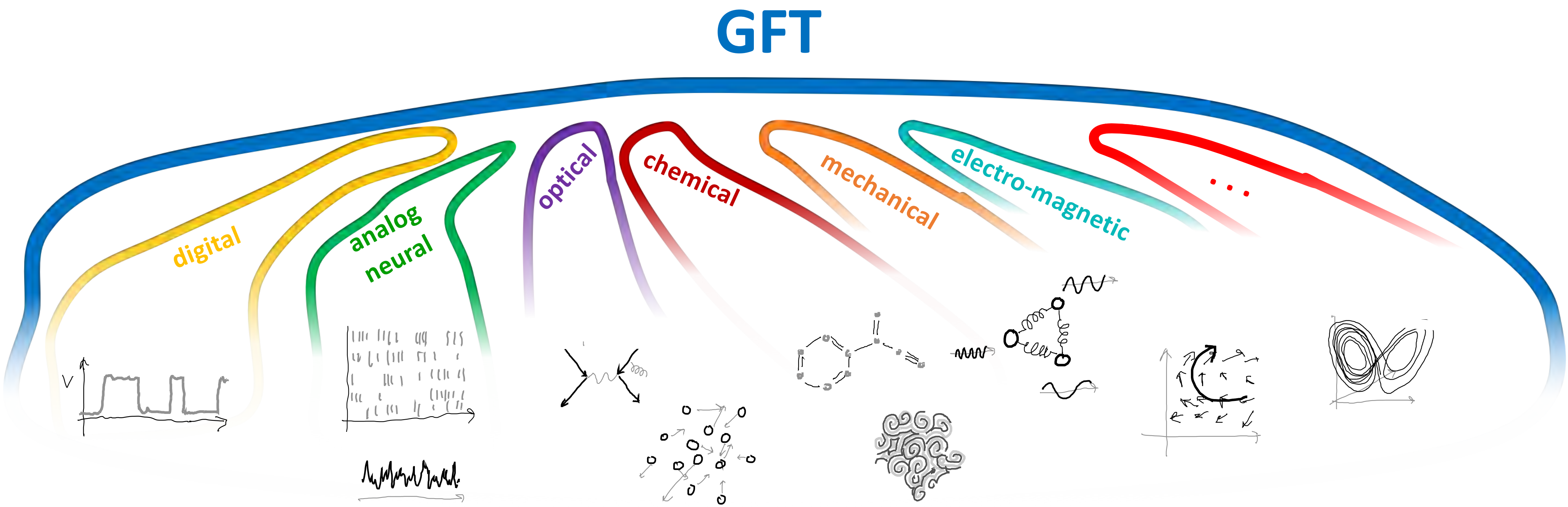


Formal interrelationships between these are known

Short intro in L. Hornischer, Semantics for Non-symbolic Computation. https://iiia.csic.es/tacl2024/

4. Back to our business

Reminder: what a GFT should cover



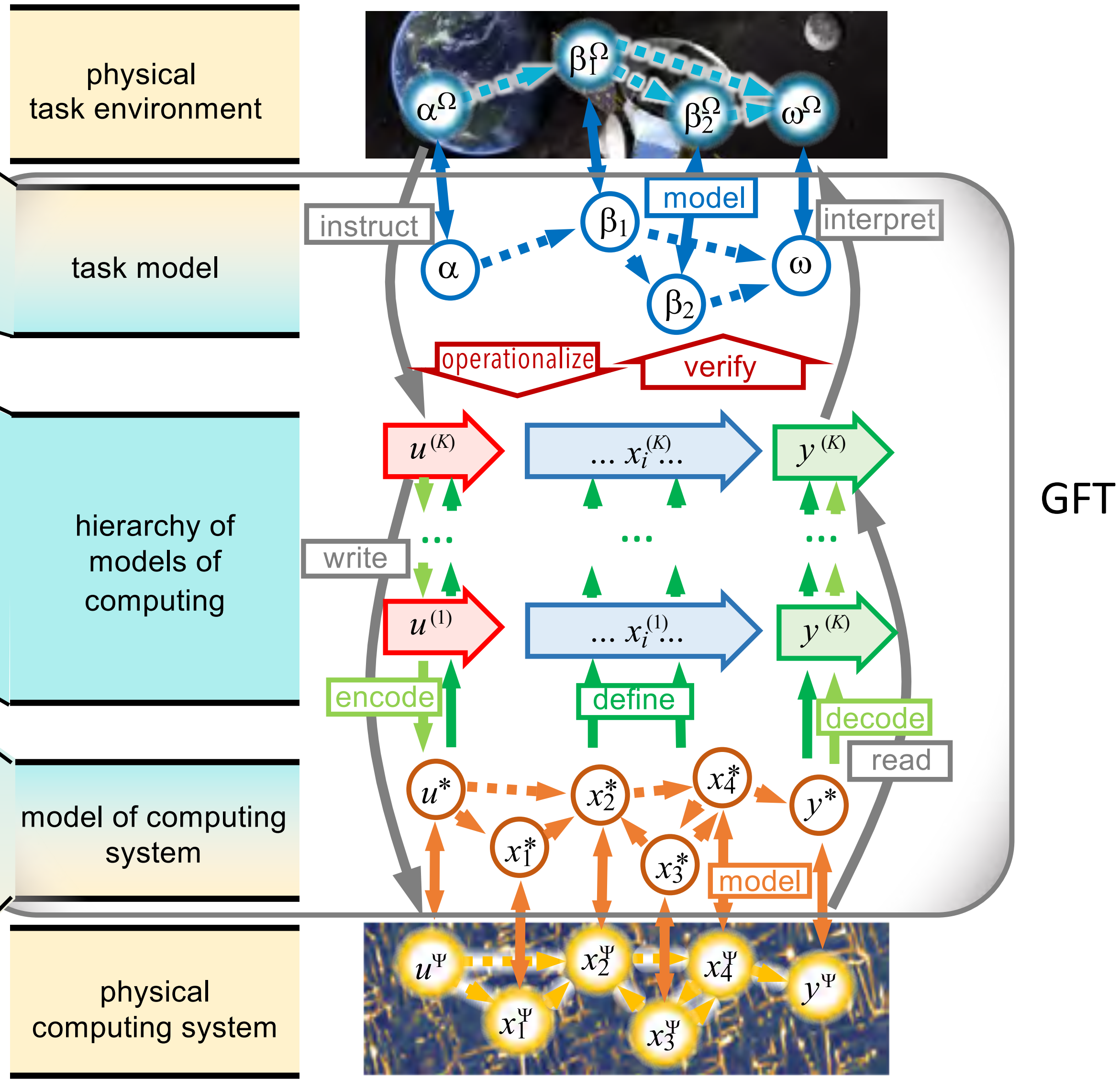
Fragments we already have for a GFT: descriptive formalisms

Besides the DC logic-based models, **not much**… some neuro-comp learning task schemes excepted, maybe

In DC: vast variety, e.g. C++, python, Turing machines, λ-calculus. Outside DC: a sizable number of **niche proposals** for non-classical ‘computing’ concepts

An variety of **well-established formalisms** for phenomenal characterizations of physical systems, e.g. from

– statistical physics
– nonlinear dynamics
– signal processing
– engineering
– network theory

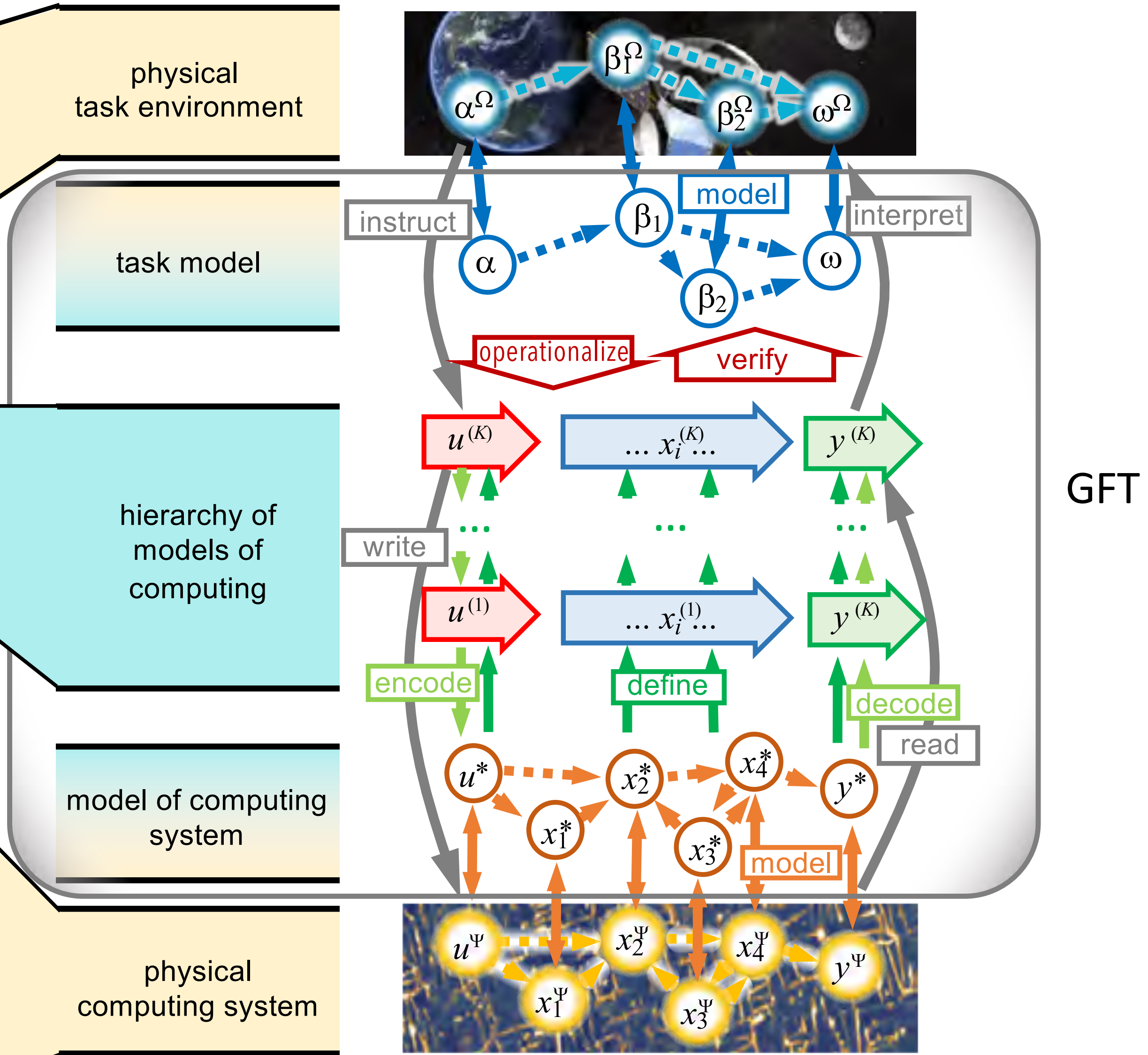


Fragments we already have for a GFT: object formalisms

Not much:
- set-theoretic S-structures and possible worlds (in classical CS)
- probability spaces $(\Omega, \mathfrak{B}, P)$

The only one: domains (in classical computer science)

Not a big choice:
- discrete state transition systems (CS)
- set-theoretic S-structures
- probability spaces $(\Omega, \mathfrak{B}, P)$
- quantitative dynamical systems formalisms like ODEs, PDEs, stoch. processes



Reminder: the confusing multiplicity of proposals for parts of a GFT

- Ultrastable systems (Ashby)
- Pattern theory (Grenander, Mumford)
- Free energy agent model (Friston)
- Reservoir computing (Maass, Jaeger)
- Stochastic computing (von Neumann)
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- Wolfram physics (Wolfram)
- Causal sets (Sorkin)
- Commuting diagrams (Horsman, Stepney)
- Neuromorphic compilation hierarchy (Zhang et al)

What we need for a GFT (opinion)

semantic 'object' models

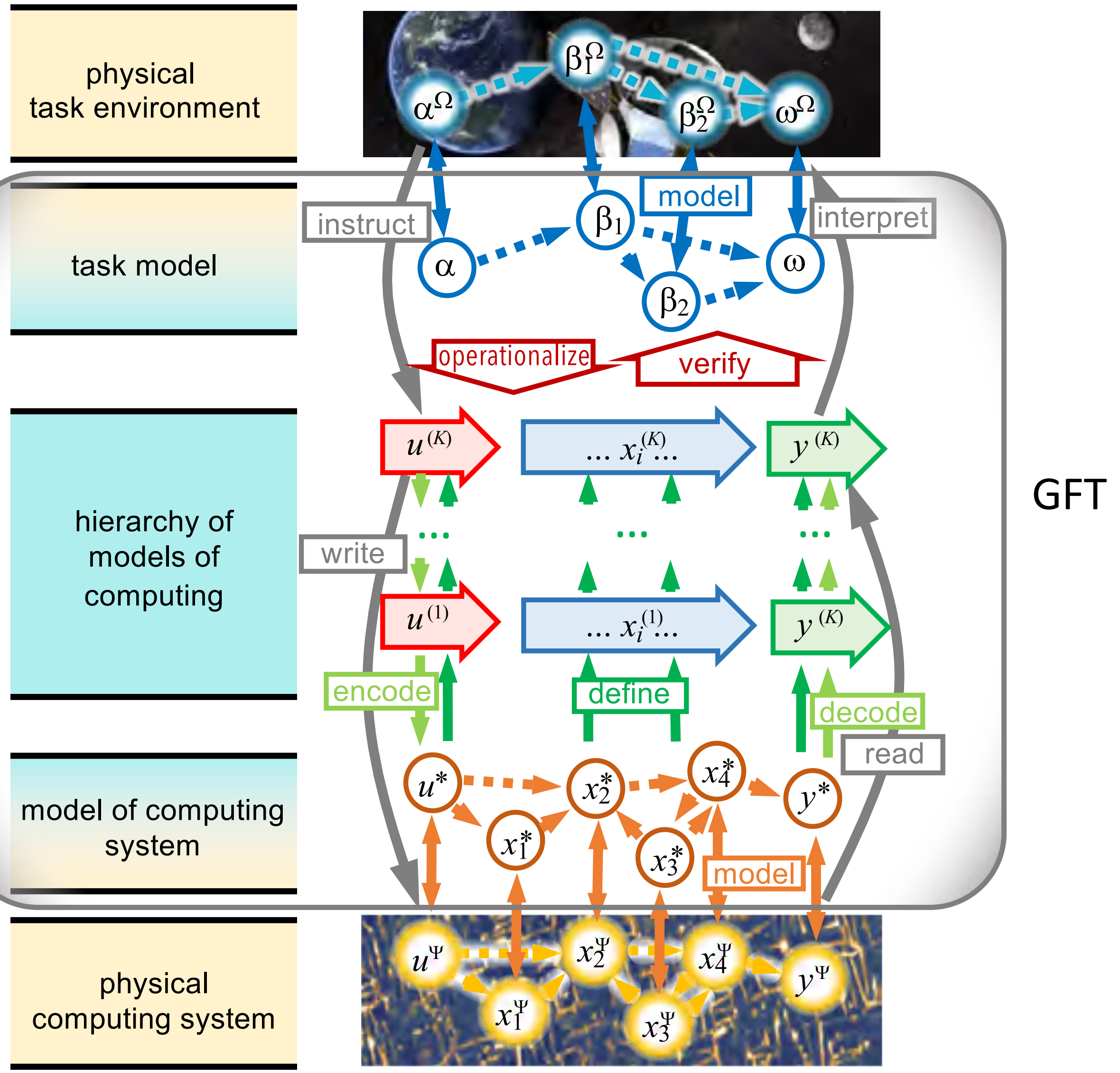
An information-oriented mathematical 'universe model'
- for tasks, processes and substrates alike
- current lead[4]: domains with spatio-temporal structure

descriptive languages

1. A **generalized view on 'computing' tasks**[1]
   - open-ended, lifelong, interactive, user collectives
   - descriptive task formalisms

2. A **generalized view on 'computing' processes**[2]
   - multiscale spatiotemporal
   - coupled into evolving task environment and physical substrate

3. A **generalized view on 'computing' substrates**[3]
   - digital, neuromorphic, or 'in-materio'



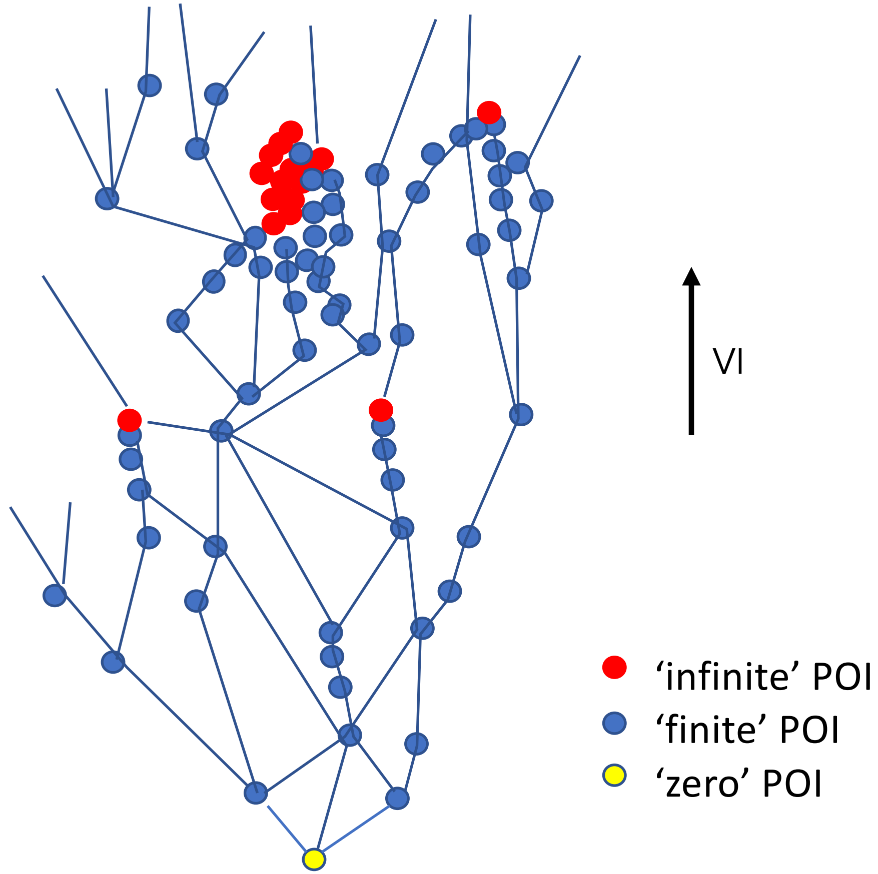
[1] PhD project Steven Abreu [2] Jaeger/Noheda/v.d.Wiel 2023 [3] ?? [4] ongoing work

5. Domains as point of departure for GFT object universes

- D. S. Scott, *Outline of a Mathematical Theory of Computation*. Technical report, Oxford Univ. 1970 https://www.cs.ox.ac.uk/files/3222/PRG02.pdf
- G. Gierz, K. H. Hofmann, K. Keimel, J. D. Lawson, M. W. Mislove & D. S. Scott, *Continuous lattices and domains* (Encyclopedia of Mathematics and its Applications, Vol. 93). Cambridge University Press, 2003 (590 pp.)

Domains: an intuitive intro

- From states of a computation process on can extract pieces of information (POIs, *my wording*) $A, B, \ldots$ by observations / decodings
- These POI's can be partially ordered, $A \leq B$ meaning that B contains more information than A
- Gives a partial order $(X, \leq)$, X the set of POIs
- Some POIs can be represented and handled finitely / effectively
  - for instance finite-decimal numbers like 0.235
  - finite definition for π
- Other POIs contain 'infinite' information, like an infinite random sequence 0.23545634…
- A goal of domain theory: full math story of how 'infinite' POIs can be approximated by 'finite' ones
- Leads to (Scott) topologies on partial orders, making $(X, \leq)$ a domain



- POIs can be immensely complex, e.g. finite or infinite sets of
  – data structures
  – logic formulas
  – graphs
- 'Computational' operations: continuous functions between domains

Why consider domains as semantic object formalism for a GFT?

A three-step argument

1. All tasks, computational processes, and physical 'computing' machines can be seen as dynamical systems
   - dynamics might end with 'stopping', i.e. terminating trajectories admitted

2. 'Computing' $\approx$ 'information processing'. Thus consider states of dynamical systems as denoting 'pieces of information'
   - as processing goes on (i.e. dynamical system evolves), information may grow, shrink, change

3. Hence, interpret states of dynamical systems as the 'pieces of information' elements $x =$ ● ● ● in domains $(X, \leq)$
   - state transitions f in the dynamical system become (Scott-)continuous functions $[\![f]\!]: X \to X$, i.e. 'update rules' of information states

A simple example (from Levin Hornischer 2019, 2021a)

L. Hornischer. *Toward a logic for neural networks*. In Sedlár, I. and Blicha, M. (eds). The Logica Yearbook 2018. London: College Publications, pp. 133–148, 2019
https://pure.uva.nl/ws/files/63009239/Logica2018_Hornischer.pdf

L. Hornischer, *The logic of information in state spaces*. The Review of Symbolic Logic 14(1) 155-186, 2021a

L. Hornischer, *Dynamical Systems via Domains: Toward a Unified Foundation of Symbolic and Non-symbolic Computation*. PhD thesis, Institute for Logic, Language and Computation, U Amsterdam, 2021b

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Munich Center for
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Consider a discrete-time, deterministic dynamical system $f: S \to S$

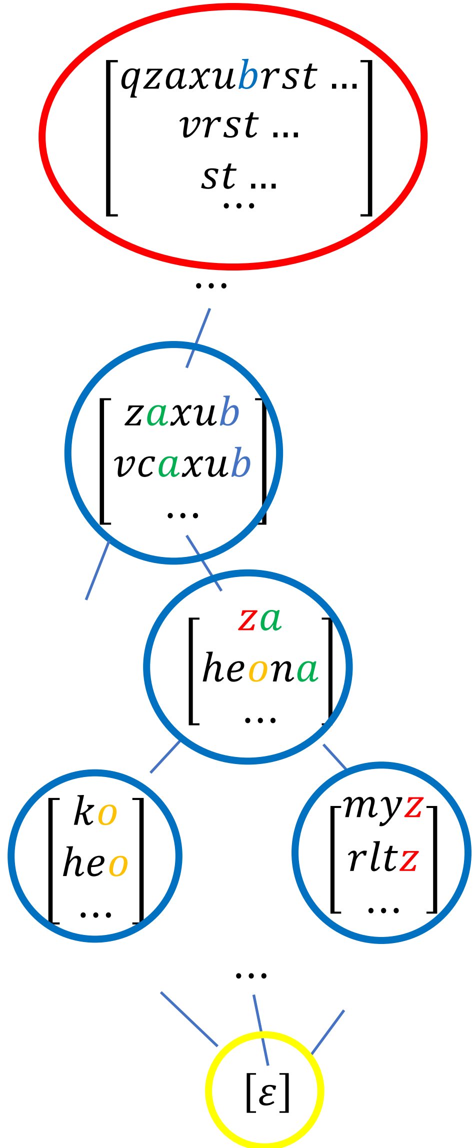
- Consider the set T of all finite and infinite trajectories t
- Define equivalence relation on T by

$$t \equiv t' \text{ iff } \begin{cases} t, t' \text{ are both finite and end in same state, or} \\ t, t' \text{ are both infinite and share a state} \end{cases}$$

- Define partial order on equivalence classes by

$$[t] \leq [t'] \text{ iff } \begin{cases} t \text{ is finite and its last state occurs in some element of } [t'] \\ \text{or} \\ t \text{ is infinite and equivalent to } t' \end{cases}$$

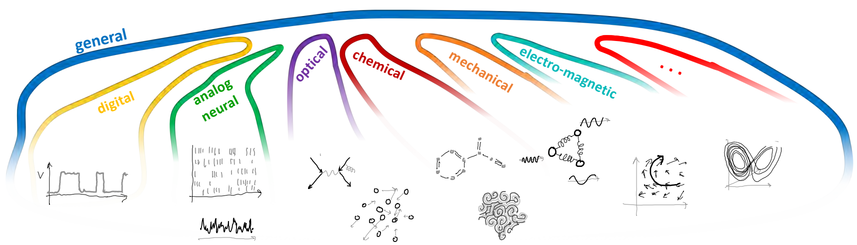
- Then $(T/_{\equiv}, \leq)$ is a domain, with 'infinite' limit POIs ● the $[t]$ where t is infinite



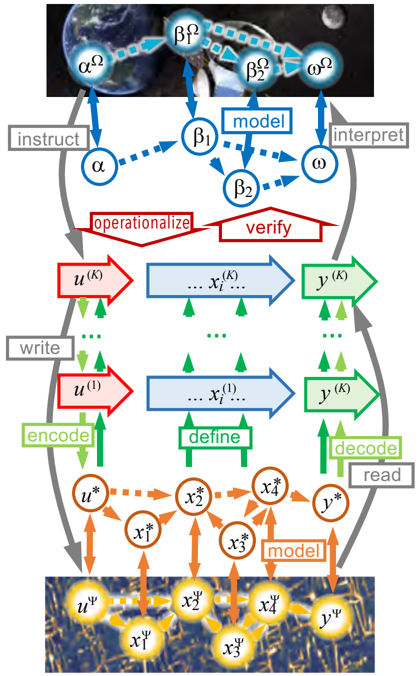
- Hornischer's PhD thesis treats the vastly more general case of measure-preserving maps f

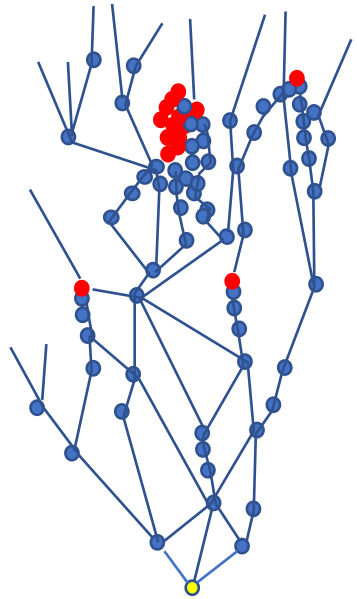
6. Summary

Needed: a GFT as interdisciplinary coordination backbone for **studying 'computing' systems of all sorts**



A GFT is not a single, simple mathematical model but **a textbook corpus of interrelated sub-theories**





Like in digital computing theory, we must model

- 'computing' processes,
- tasks,
- physical substrates and 'machines'

'Computing' is processing information that is **meaningful for a user / observer**. 'Computing' is always 'computing about'

- No useful theory of 'computing' without modeling the semantics of computing processes

A necessary subtheory of a GFT: a mathematical formalism for **characterizing semantic 'referent object' universes**

- whose structure reflects the information we have about them,
- whose dynamics reflects the information processing we do in 'computing about them'

Image sources

Desktop computer: https://www.bhphotovideo.com/c/product/736195-REG
Centrifugal governor: Wikimedia commons – drawing from from W. Ripper: *Heat Engines*. Longman, London 1909
Brain: Wikimedia commons - NIH
Hand: Wikimedia commons - Evan-Amos
Eye: Wikimedia commons – user *rapidreflex*
Scientist talking: Wikimedia commons – DataBase Center for Life Science (DBCLS)
Computer action with hand: https://www.youtube.com/watch?v=QvAxrnodq8w
Golden chain: Wikimedia commons - user Bondigoldwiki
Chess move: Wikimedia commons - user Walter J. Pilsak
Scientist with notebook: Wikimedia commons – DataBase Center for Life Science (DBCLS)
Real-World circle picture: http://www.kim-mckellar.com/round-world-card.html (now inaccessible)
Sunset dance: https://www.123rf.com/free-images/
Ribbon circle: Wikimedia commons – user Cuberoottheo
Hornischer portrait: - his homepage