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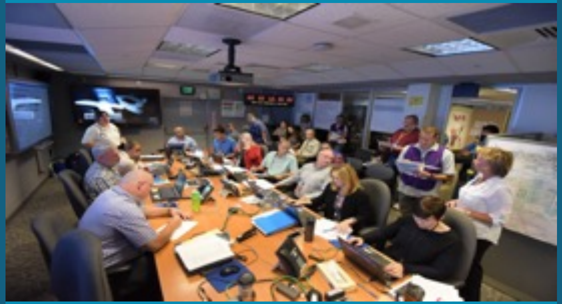
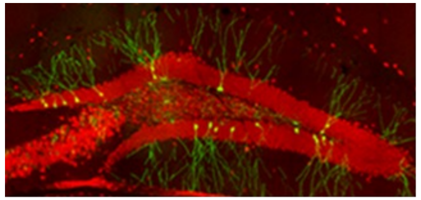
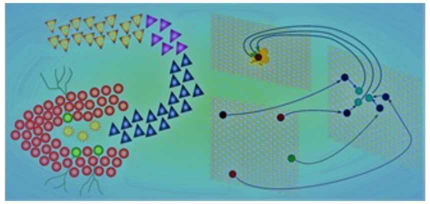
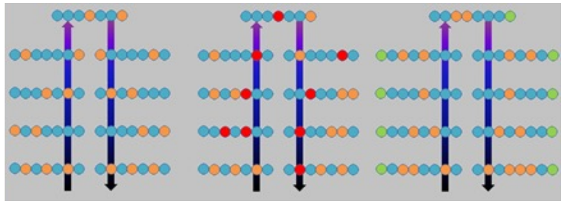
Localization through Grid-based Encodings on Digital Elevation Models









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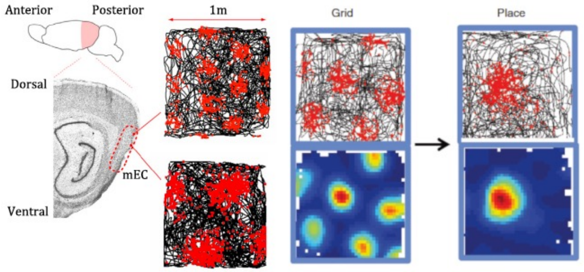
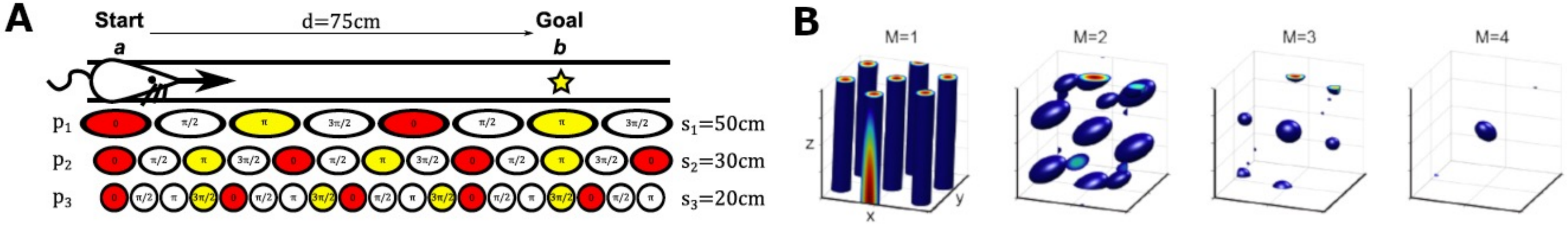
Introduction and Motivation

- Robust sensor-aided localization
  - Applications to strategies for intelligent navigation directly from on-board sensors in challenging environments and/or with resource constraints (e.g. terrain relative navigation)
  - We take a neuro-inspired model of distributed grid-based computation and apply it in the context of navigation-based datasets (e.g. digital elevation models)
- Neural inspiration from grid cells
  - Hippocampal representation of space using grid cells (in addition to place cells)
  - Characterized by a periodic, hexagonal tiling with different spatial scales, orientations, and offsets
  - Intersection of multiple grid modules can be decoded yield unique locations



Grid cell activations of the rat hippocampus collected over square arena [*Moser et al. Place Cells, Grid Cells, and Memory*]

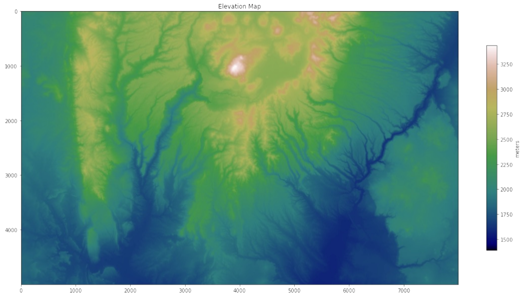


Intersection of multiple grid modules encode locations in 1D (a) and 3D (b) space [*(a) Bush et al. Using Grid Cells for Navigation; (b) Klukas et al. Flexible representation of higher dimensional cognitive variables with grid cells*]

Grid Cell Activations Over a Map

- Overlaying grid cell activations onto digital elevation models (DEMs) provides a grid-based representation of locations

Sample DEM map (area around Albuquerque)



$$k_1 = \frac{4\pi\lambda}{\sqrt{6}} \times \left(\left(\cos(\theta + \frac{\pi}{12}) + \sin(\theta + \frac{\pi}{12})\right) \times (x - \varphi_x) + \left(\cos(\theta + \frac{\pi}{12}) - \sin(\theta + \frac{\pi}{12})\right) \times (y - \varphi_y) \right)$$

$$k_2 = \frac{4\pi\lambda}{\sqrt{6}} \times \left(\left(\cos(\theta + \frac{5\pi}{12}) + \sin(\theta + \frac{5\pi}{12})\right) \times (x - \varphi_x) + \left(\cos(\theta + \frac{5\pi}{12}) - \sin(\theta + \frac{5\pi}{12})\right) \times (y - \varphi_y) \right)$$

$$k_3 = \frac{4\pi\lambda}{\sqrt{6}} \times \left(\left(\cos(\theta + \frac{3\pi}{4}) + \sin(\theta + \frac{3\pi}{4})\right) \times (x - \varphi_x) + \left(\cos(\theta + \frac{3\pi}{4}) - \sin(\theta + \frac{3\pi}{4})\right) \times (y - \varphi_y) \right)$$

$$G = \frac{2}{3}\left(\frac{k_1 + k_2 + k_3}{3} + .5 \right)$$

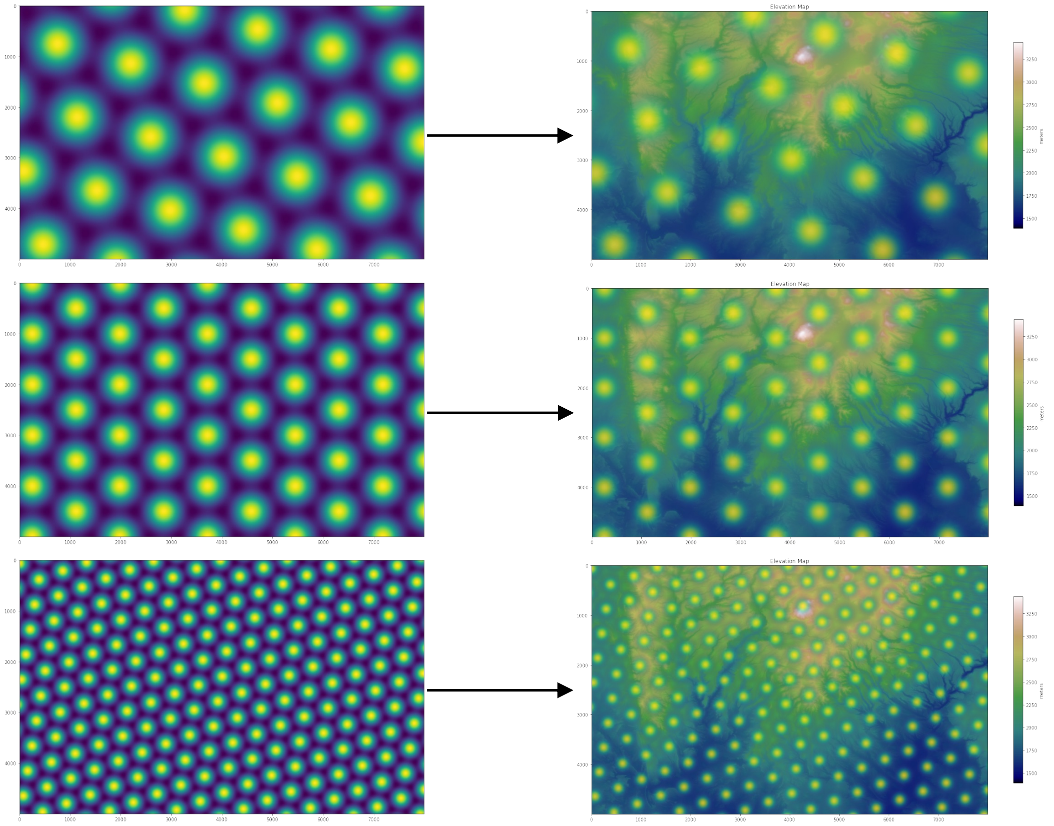
Equations for grid cell activations
[*Solstad et al. From grid cells to place cells: a mathematical model*]

Sample grid cells of distinct periods, orientations, and offsets overlaid on the same elevation map. Centroids correspond to locations with high activation.

$\lambda = 1500\text{px}$
$\theta = \frac{\pi}{4}$
$\phi = (\pi, \pi)$

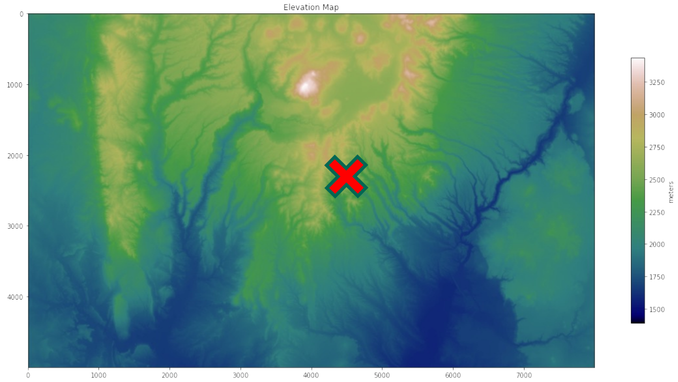
$\lambda = 1000\text{px}$
$\theta = \frac{\pi}{6}$
$\phi = \left(\frac{\pi}{2}, 0\right)$

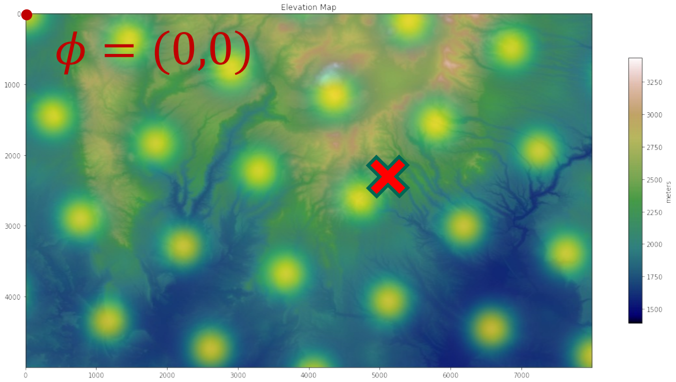
$\lambda = 500\text{px}$
$\theta = \frac{\pi}{9}$
$\phi = \left(0, \frac{\pi}{4}\right)$

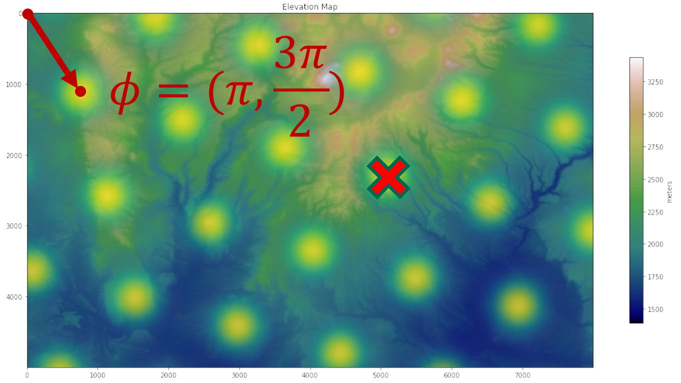


Representing Locations Using Grid Modules

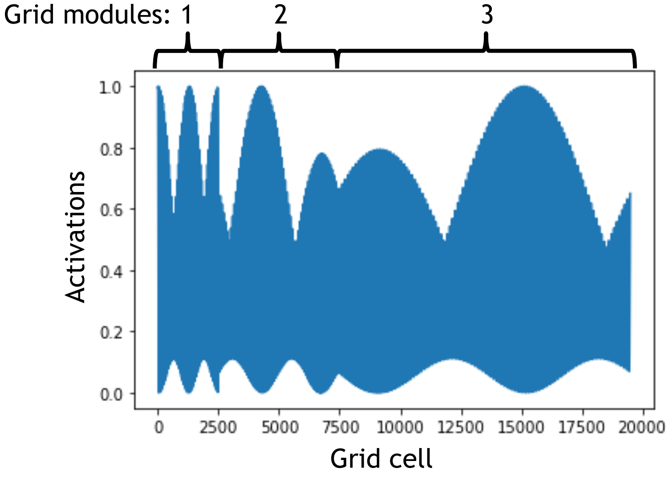
- Refinement of grid-based representation from individual grid cell activations to grid module phase codes enables greater representation and more tractable computation







Grid modules defined by shared period and orientation, whereas their "phase" determines their offset w.r.t. a reference point (e.g. $\phi = (0,0)$)



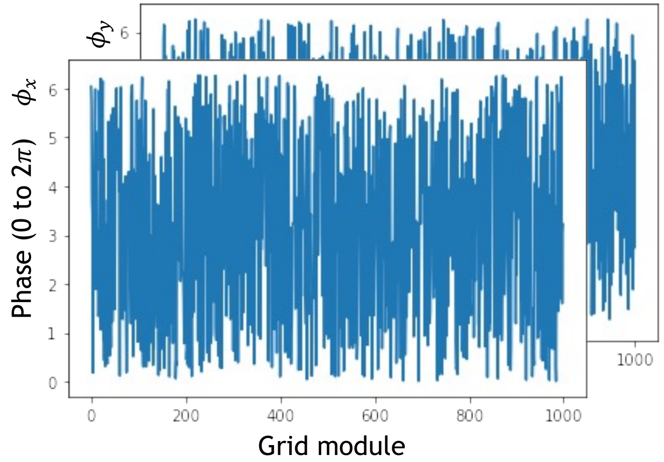
Representation of a location (x,y) using:

- Grid cell activations (left)

$$(x, y) \rightarrow \{a^1_{0,0}, a^1_{\Delta\phi_x,0}, a^1_{\Delta\phi_x,\Delta\phi_y}, \ldots, a^m_{k\Delta\phi_x,k\Delta y}\}$$

- Grid module phases (right)

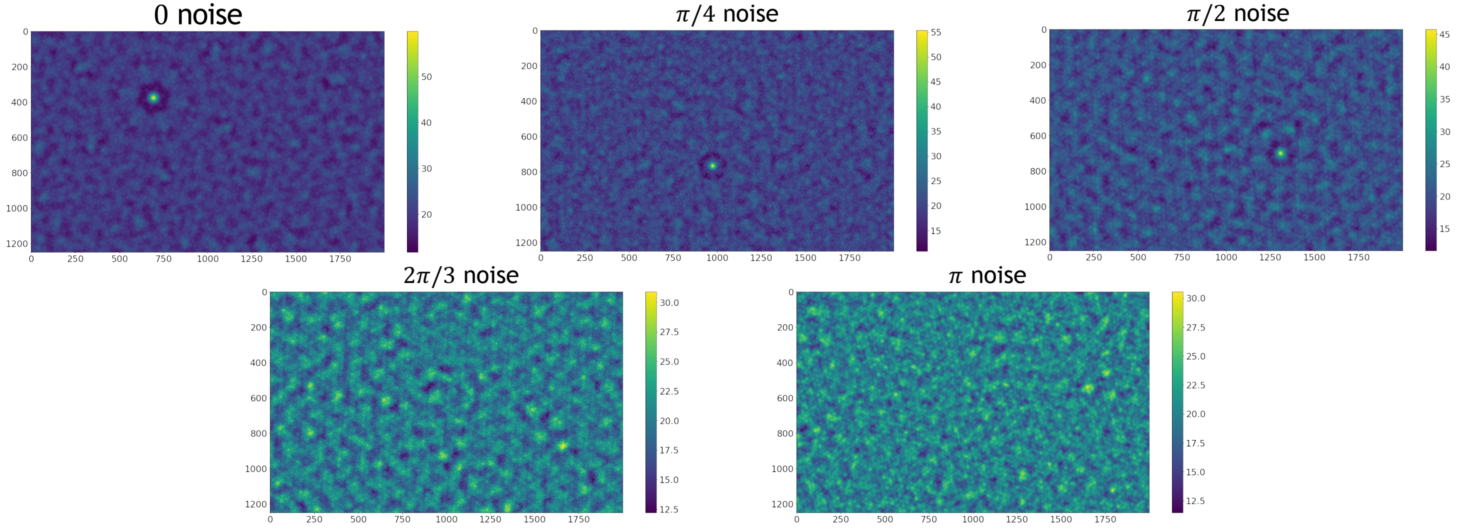
$$(x, y) \rightarrow \{\phi^1_x, \phi^1_y, \phi^2_x \ldots, \phi^m_y\}$$

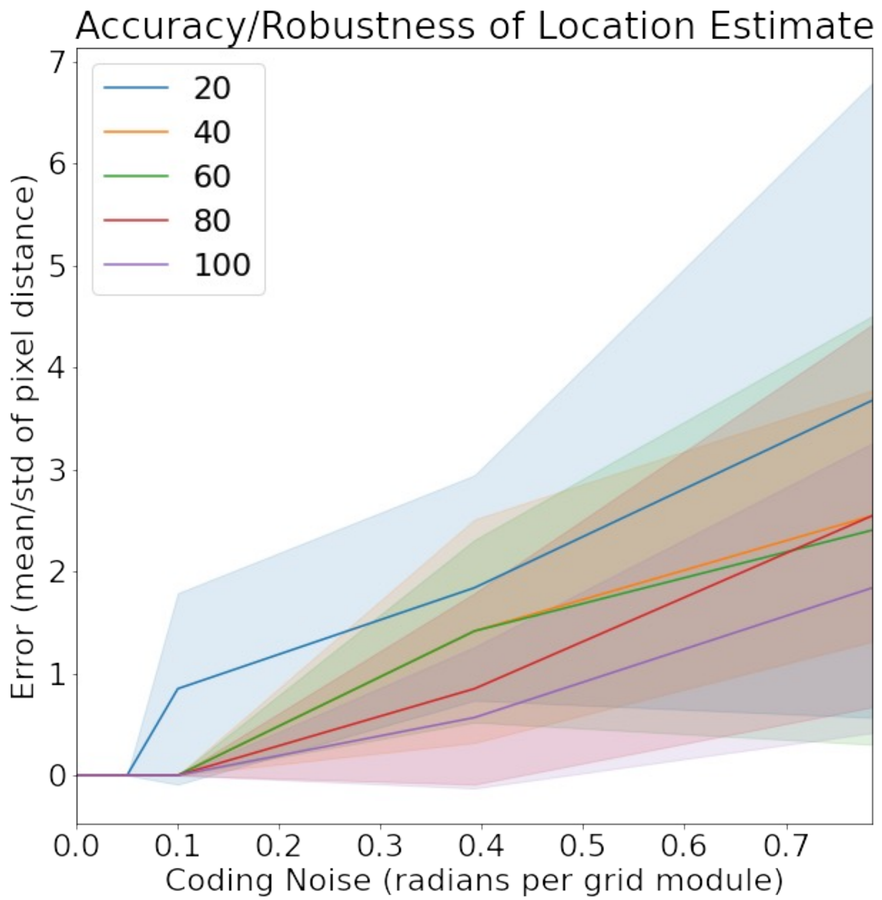


Decoding Accuracy and Robustness

- Redundancy through the use of multiple grid modules results in strong accuracy and robustness properties in location estimates
- Decoded coincidence maps maintain strong signal to noise ratio of the intersection point despite coding noise (e.g. zero-centered, uniform)

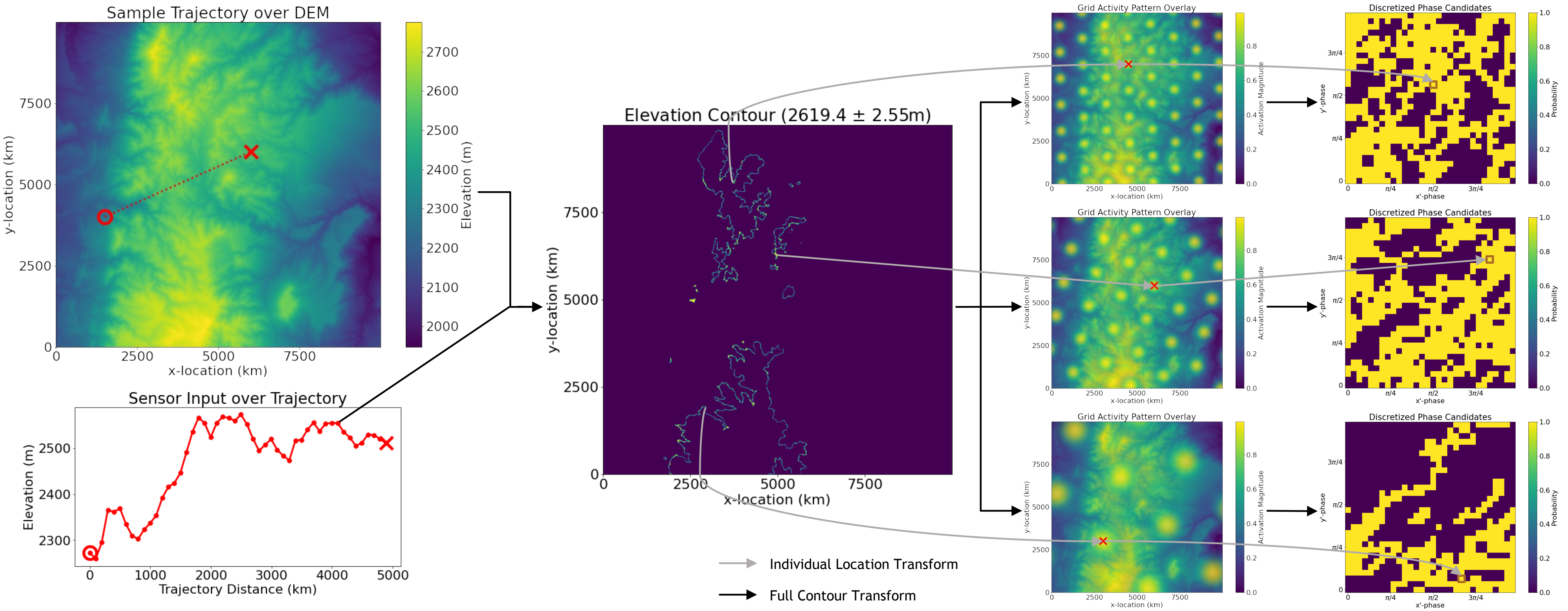
Sample coincidence maps of different locations decoded using 60 grid modules for increasing levels of coding noise





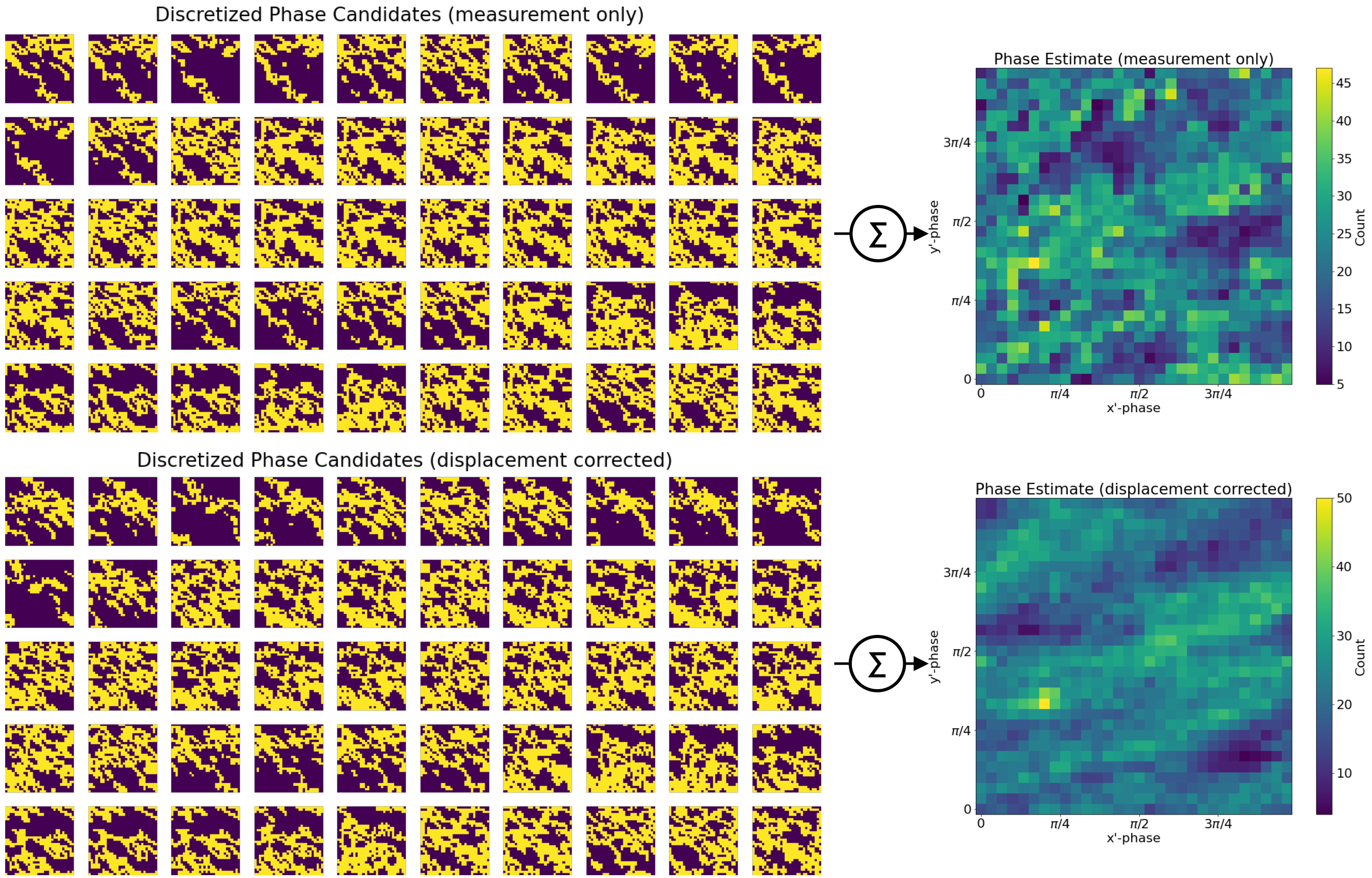
Encoding to Grid Module Phase Codes

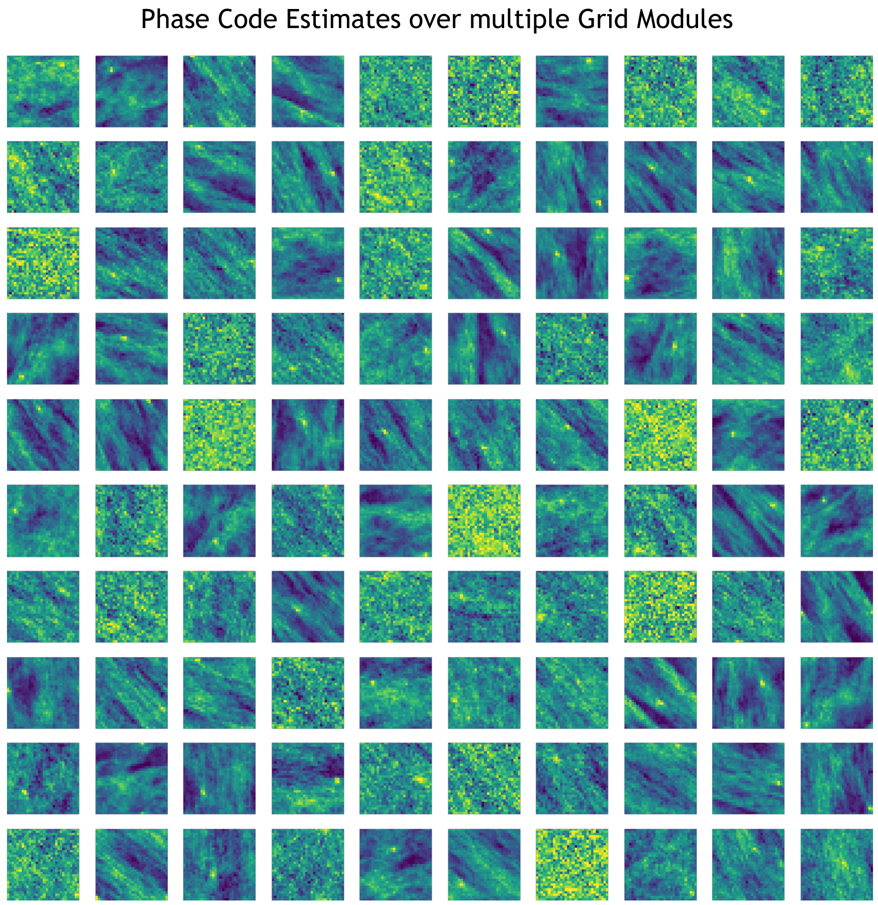
- We perform a distributed correlation over the grid modules from sensor inputs
  - Here, elevation contours are transformed onto possible phase code candidates



Encoding to Grid Module Phase Codes

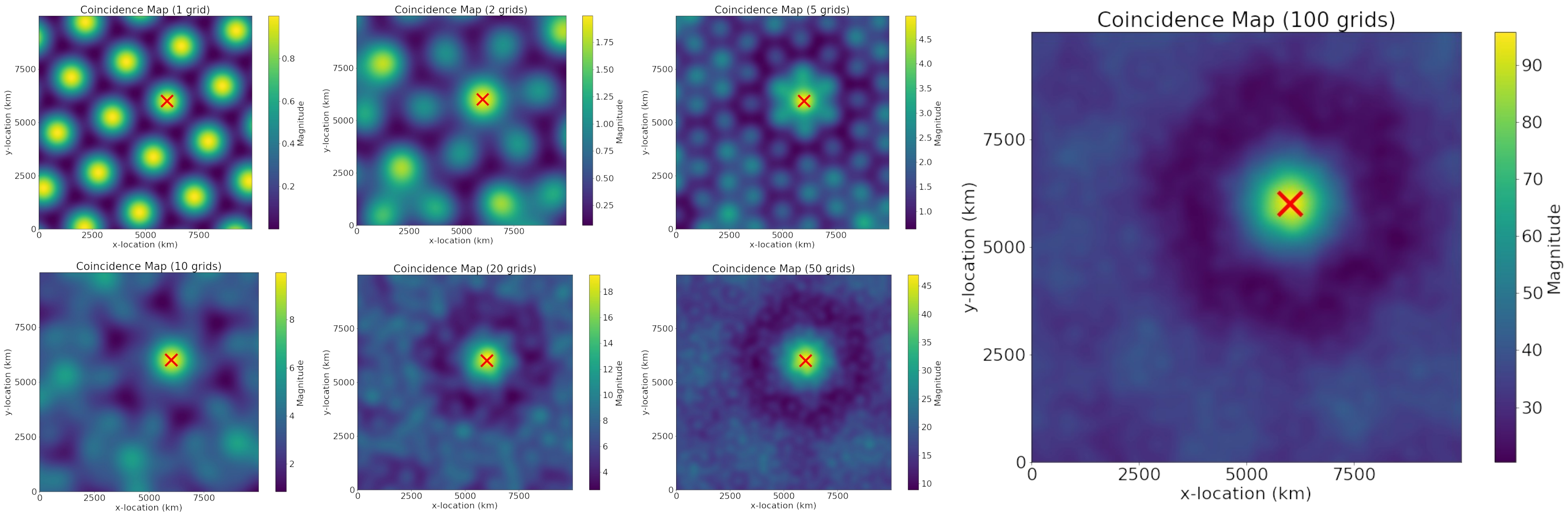
- Spatial displacement corresponds to phase shifts and can be integrated with respect to a reference time/location from multiple measurements





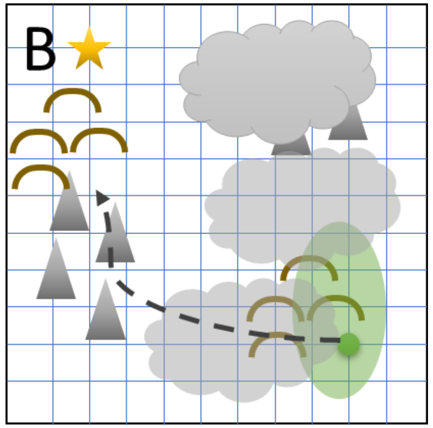
Decoding to the Location Estimate

- By summing the grid cell activations corresponding to grid module phase codes, we can compute a "coincidence map" to find where they may uniquely intersect
- This computation is scalable, where only a subset of grid modules is required for successful decoding, and grid modules can be encoded/trained independently



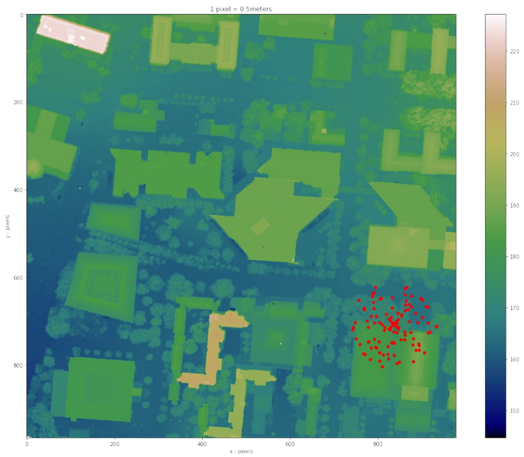
Summary and Future Work

- We showed simulations on navigation-based datasets (DEMs) applying our neuro-inspired model of distributed grid-based computation to localize from a set of elevation inputs
- Current and Future Work
  - Analysis of tradeoff spaces (e.g. computation, storage costs, robustness)
  - Learning/training phase candidates from data (e.g. mapping part of SLAM)
  - Adaptation of localization algorithm to different datasets, sensor and noise models, and integration with filters (e.g. EKF update)

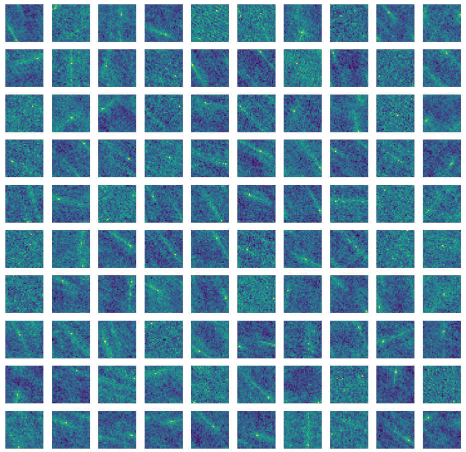


Goal: leverage neuro-inspired strategies in support of intelligent navigation

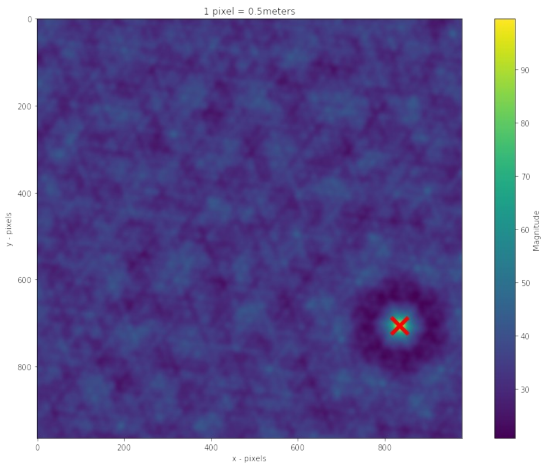
Sample Measurements over Point Cloud



Phase Code Estimates over multiple Grid Modules



Coincidence Map from Estimated Phase Code



Single location, randomly sampled measurements: grid-based localization algorithm applied to a rasterized point cloud dataset (UT-Austin)

Backup: Representing Locations Uniquely

- To represent locations uniquely, we need the phase code dimensions to be orthogonal
  - This is achieved by performing an affine/shear-like transformation per grid module
  - With period and orientation fixed per grid module, the phase corresponds to the offset of the corresponding grid cell that is maximally active at the encoded location
  - This is computed using the modulo operator in the orthogonalized space

Affine transform
$$(x_i', y_i') = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i + \frac{\pi}{6}) \\ \sin(\theta_i) & \cos(\theta_i + \frac{\pi}{6}) \end{bmatrix} (x, y)$$

Modulo operation
$$\phi_x^i = x_i' \bmod \lambda_i$$
$$\phi_y^i = y_i' \bmod \lambda_i$$

Sample grid cell activation transformed into orthogonalized space (and thresholded image for clarity)