Towards fractional order dynamics neuromorphic elements

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NICE, La Jolla, CA April 24th, 2024

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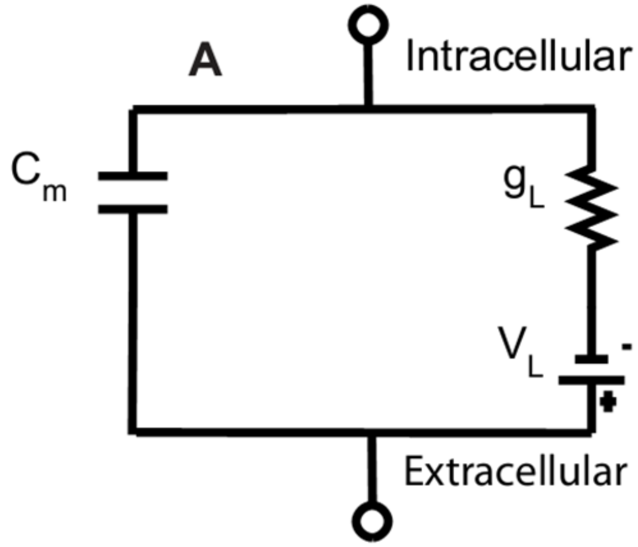
[matrix] The UTSA AI Consortium for Human Well-Being

Challenge: A model that is used as theory

- The leaky integrate-and-fire

$$C_m \frac{dV}{dt} = -g_m(V - V_{rest}) + I_{inj}$$

Leaky integrate-and-fire



- Unfortunately, this model has been assumed to be a foundational part of theory
- Another unfortunate assumption is that all the complexity of the brain resides in the connectivity and synaptic weights

On the role of theory and modeling in neuroscience
D Levenstein, VA Alvarez, A Amarasingham, H Azab… - Journal of Neuroscience, 2023

Development of theoretical frameworks in neuroscience: a pressing need in a sea of data
HG Rotstein, F Santamaria - arXiv preprint arXiv:2209.09953, 2022

The ideal capacitor does not exist

- The ideal capacitor is not physically possible
- Any basic electromagnetism book explains the assumptions to come up with the ideal capacitor

History dependence & power law charging

Supposons que la courbe (1) représente l'intensité du courant de charge en fonction du temp

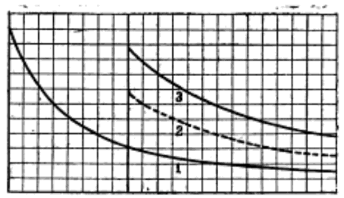


Fig. 9. — Loi de superposition : 1 première charge ; 2 deuxième charge seule (action supposée ; 3 action résulante (réelle).

$$c = \frac{a}{t^n}$$

The charging of a capacitor follows a power-law!!!!!!!!

Curie J.
Recherches sur le pouvoir inducteur spécifique et sur la conductibilité des corps cristallisés
Ann. Chim. Phys., 17 (1889), pp. 385-434

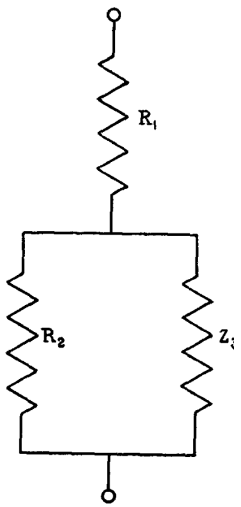
Constant phase element

ELECTRIC PHASE ANGLE OF CELL MEMBRANES

By KENNETH S. COLE

(From the Department of Physiology, College of Physicians and Surgeons, Columbia University, New York)

(Accepted for publication, April 4, 1932)



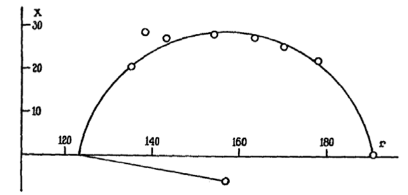


Fig. 2. Reactance vs. resistance in ohms for calf blood (Fricke and Morse)

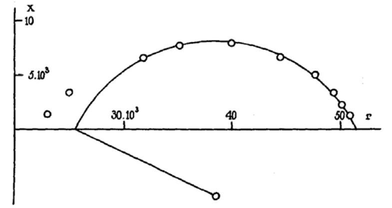


Fig. 3. Reactance vs. resistance in ohms for frog nerve (Lullies)

Fig. 1. Assumed equivalent tissue circuit

Also see Cole KS (1933) Electric Conductance of Biological Systems. Cold Spring Harb Symp Quant Biol 1: 107–116.

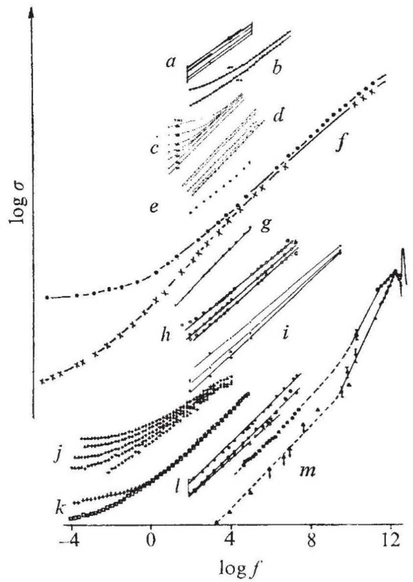
Power law & history dependence in dielectrics

Nature Vol. 267 23 June 1977

review article

The 'universal' dielectric response

A. K. Jonscher*



The mathematical object that gives constant phase and power-law behavior

- The ideal capacitor is

$$I = C\frac{dV}{dt}$$

- Which results in an exponential
- What is the mathematical structure that when solved provides a power-law and constant phase?

$$I = C\frac{d^\eta V}{dt^\eta}$$

- Take a minute to stare at the equation an realize that is has values in this range: $0 < \eta < 1$

Fractional differentiation and history dependence

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- The fractional derivative is

$$\frac{d^\eta V}{dt^\eta} = \frac{1}{\Gamma(1-\eta)} \int_0^t \frac{V'(\tau)}{(t-\tau)^\eta} d\tau$$

- This is a non-local operator that has memory
- Some people call this the intrinsic memory trace
- Thus, the systems depends on time, not just on its previous state (break down of the Markovian assumption)

Fractional differentiation, constant phase, and input whitening

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- The fractional derivative of a sine wave is

$$\frac{d^\eta A\sin(2\pi\omega t)}{dt^\eta} = (2\pi\omega)^\eta A\sin\left(2\pi\omega t + \frac{\eta\pi}{2}\right)$$

- If input follows a power-law, $A = (2\pi\omega)^{-\beta}$

$$I_n = (2\pi\omega)^{-\beta}\sin(2\pi\omega t)$$

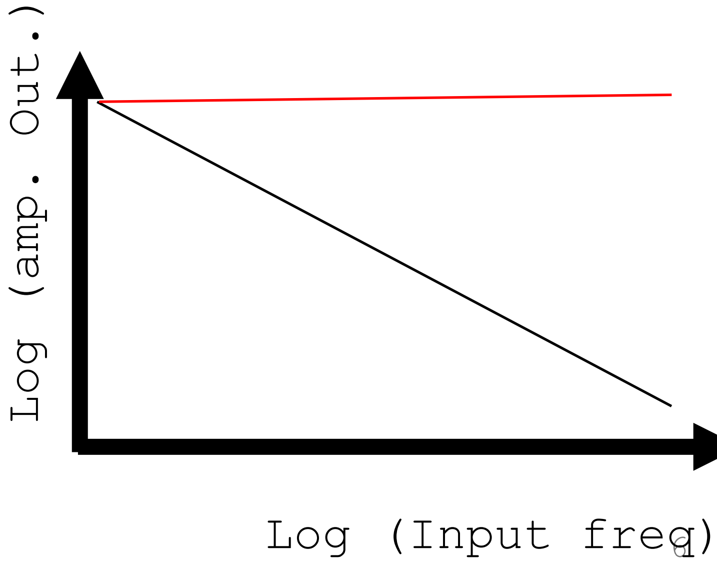
- Then

$$f = \frac{d^\eta I_n}{dt^\eta} = (2\pi\omega)^{-\beta}(2\pi\omega)^\eta \sin\left(2\pi\omega t + \frac{\eta\pi}{2}\right)$$

- If $\beta = \eta$

$$f = 1 * \sin\left(2\pi\omega t + \frac{\eta\pi}{2}\right)$$

- Optimal coding of pink noise spectra
- **Most natural signals follow pink noise**
  - **Texture**
  - **Sounds – natural, human-made**
  - **Odorants – natural plumes, food recipes**
  - **Images – color, intensity**



The sensory World has power-law statistics

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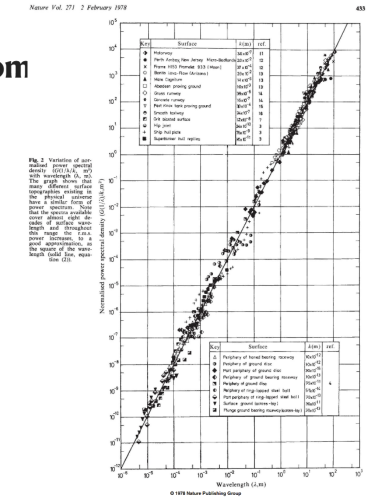
- Alternative argument: What is the mathematical transformation that optimall

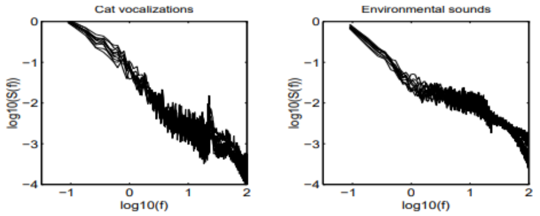
Published: 02 February 1978

Surface topography as a nonstationary random process

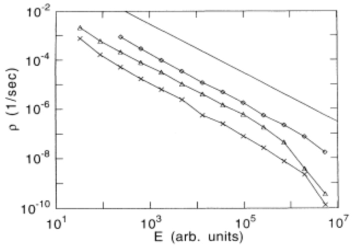
R. S. SAYLES & T. R. THOMAS

Nature 271, 431–434 (1978) | Cite this article

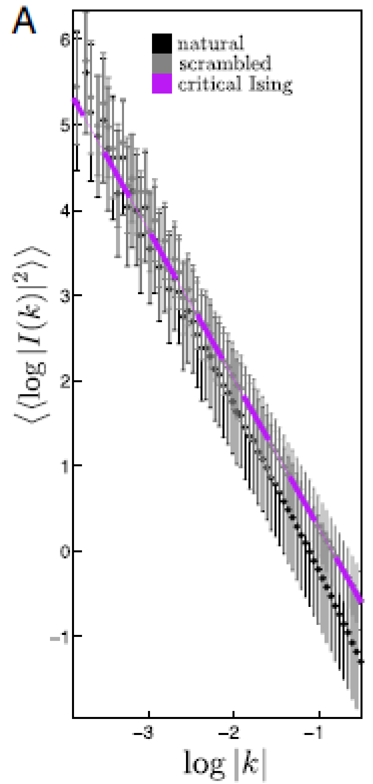




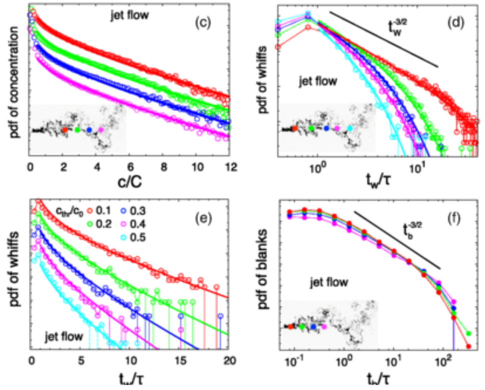
Sounds
Attias and Schreiner (1996) Advances in neural information processing systems



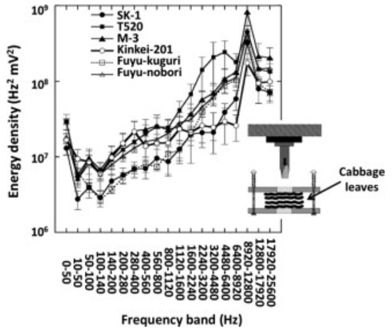
Crumpled plastic sheet
Kramer and Lobkovsky (1996) Physical Review E



Natural images
Saremi & Sejnowki PNAS

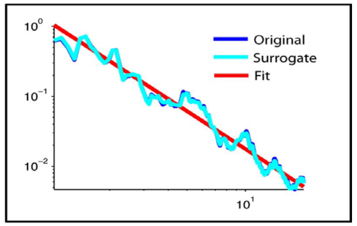


Odor concentration
Celani, Villermaux et al. (2014) Physical Review X



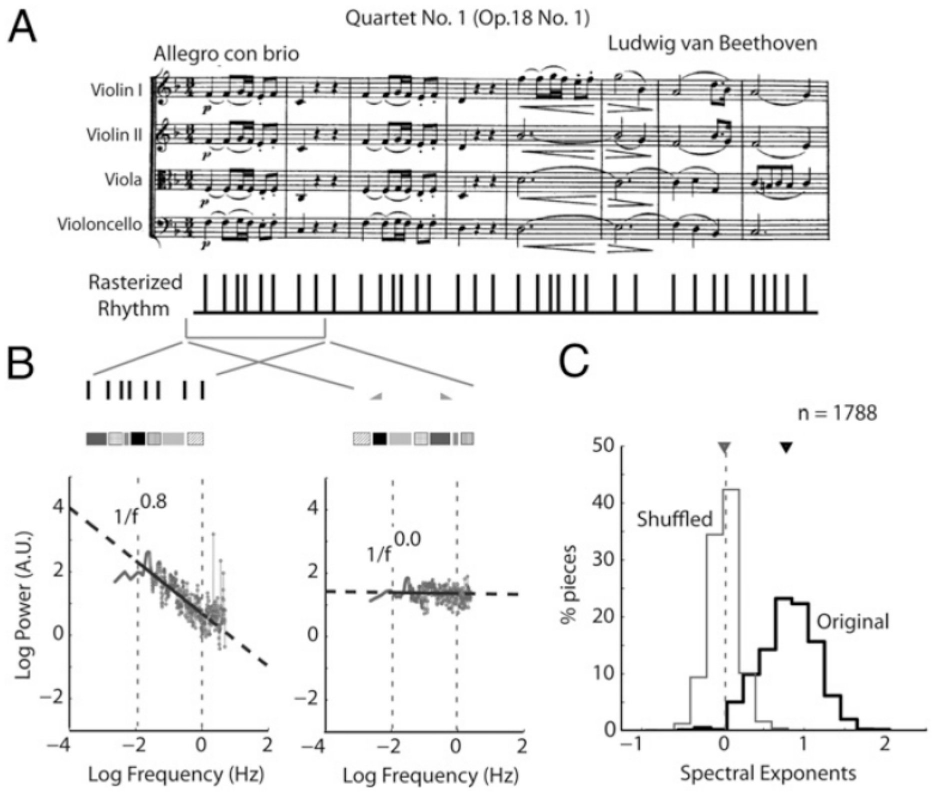
Food texture
Chen and Opara (2013) Journal of Food Engineering

log(Power) ~ –1.8 log(f)

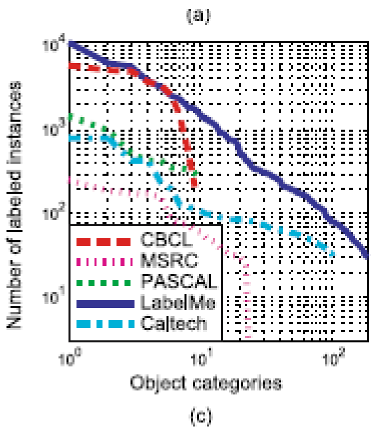


The sound of water
Geffen et al, Front Int. Neuro, 2011

Power-law statistics in human signals

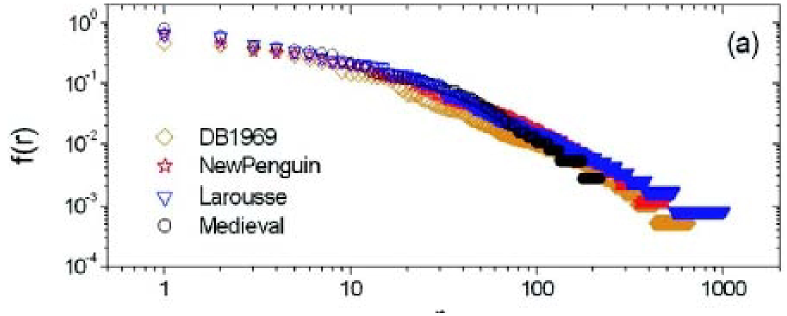


Levitin, Chordia, Menon, PNAS 2011



Naming things

Russel et al, In J Comput Vis 2008



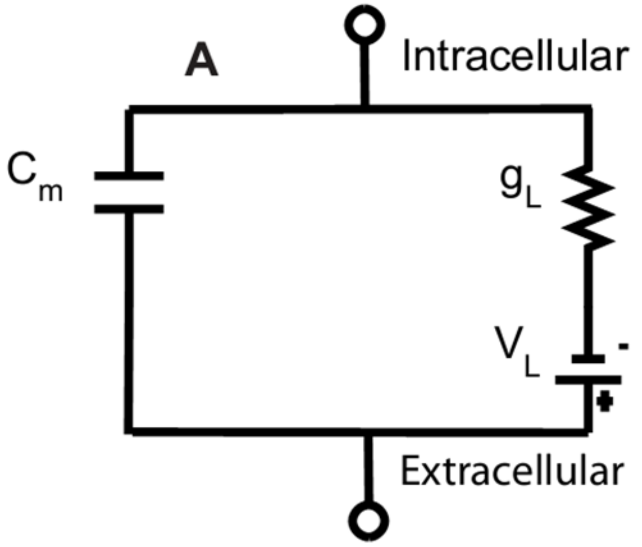
Food recipes

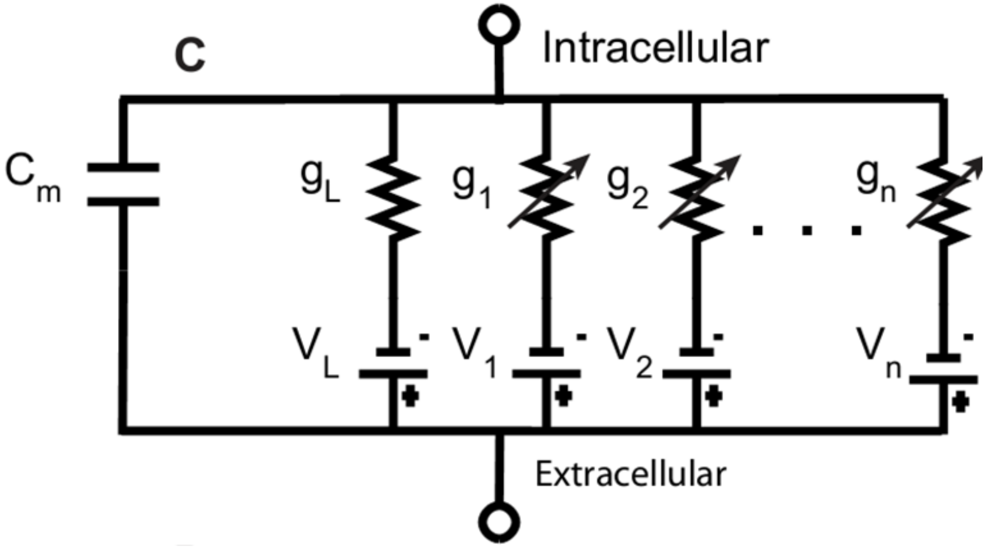
Kinouchi, Diez-Garcia et al. (2008) New Journa [matrix] The UTSA AI Consortium for Human Well-Being

Computational advantages of fractional neurodynamics

Another way to generate fractional order dynamics

- Alternative argument: Fractional order dynamics arises from processes outside therm
  - Diffusion equation turns into Anomalous diffusion
  - The reaction diffusion equation is the foundation to model neuronal activity





$$C_m \frac{d^\eta V}{dt^\eta} = -g_m(V - V_{rest}) + I_{inj}$$

$$C_m \frac{dV}{dt} = -g_m(V - V_{rest})$$

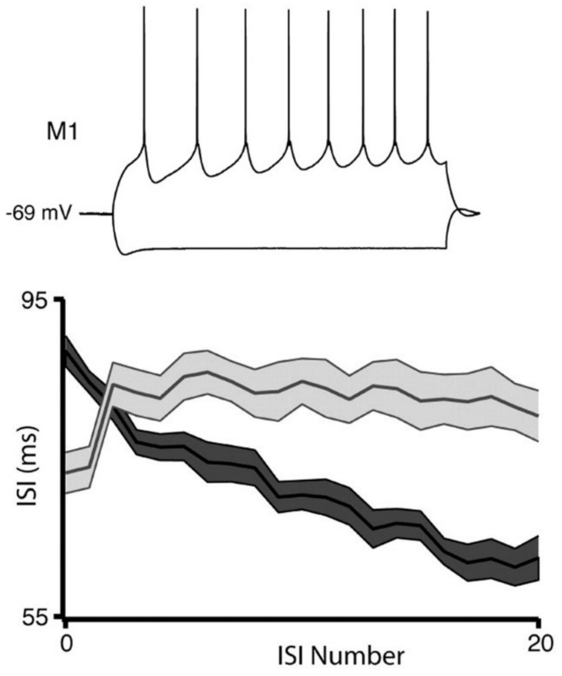
$$C_m \frac{dV}{dt} = -g_m(V - V_{rest}) - \sum_{i=1}^{N} g_i(V - V_i)$$

$$g_i = m^a n^b z^c$$

$$\frac{dm_i}{dt} = \alpha(1 - m_i) - \beta m_i$$

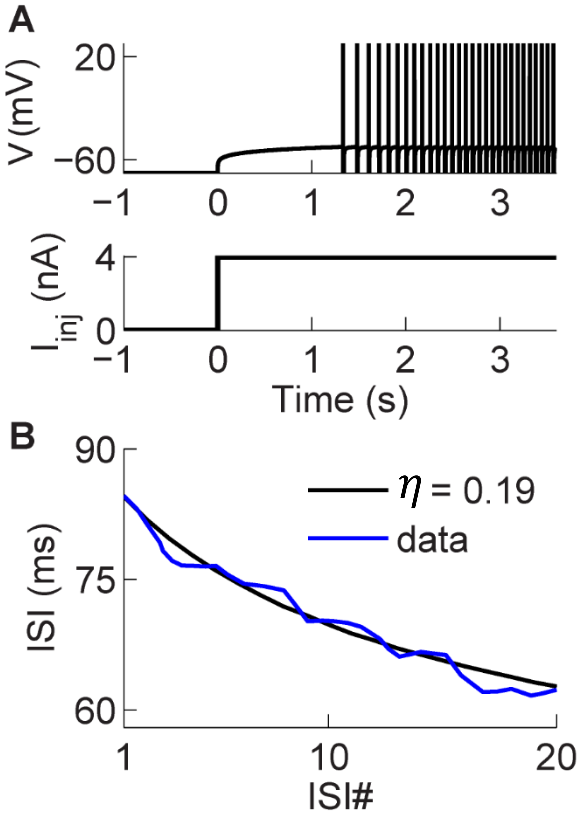
Spiking acceleration in L5 pyramidal cells

Experiments



Miller M N et al. J. Neurosci. 2008;28:13716-13726

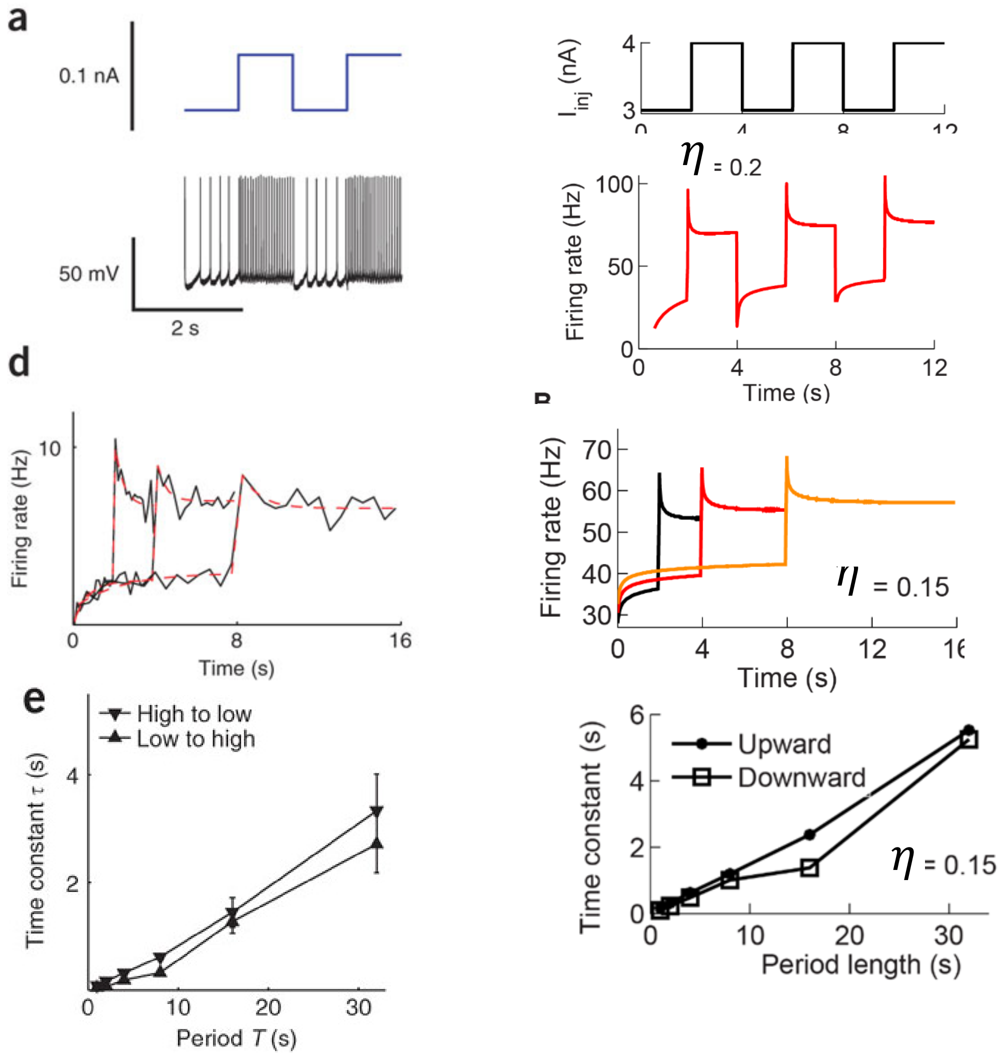
Fractional model



Teka et al., PLoS Comp. Bio 2014.

History dependence in L2/3 pyramidal cells

Experiments | Fractional model



And several other results

Teka, W., T. M. Marinov and F. Santamaria (2014). "Neuronal spike timing adaptation described with a fractional leaky integrate-and-fire model." PLoS computational biology **10**(3): e1003526.

From membrane voltage to spiking

- The fractional order is in voltage
- Does spike time or firing rate have fractional order properties?
  - Coding strategies of neurons

$$C_m \frac{d^\eta V}{dt^\eta} = -g_m(V - V_{rest}) + I_{inj}$$

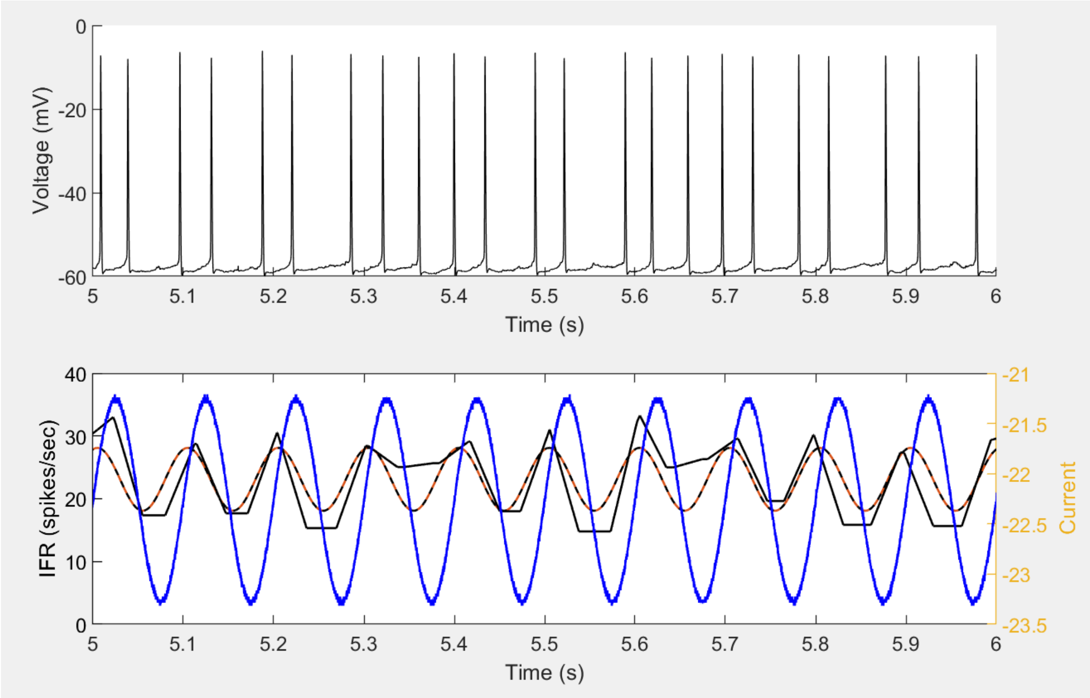
Is the fractional order reflected in the firing rate properties?

$$f \sim \frac{d^\eta I_n}{dt^\eta} \; ?$$

Evidence of fractional differentiation of firing rate

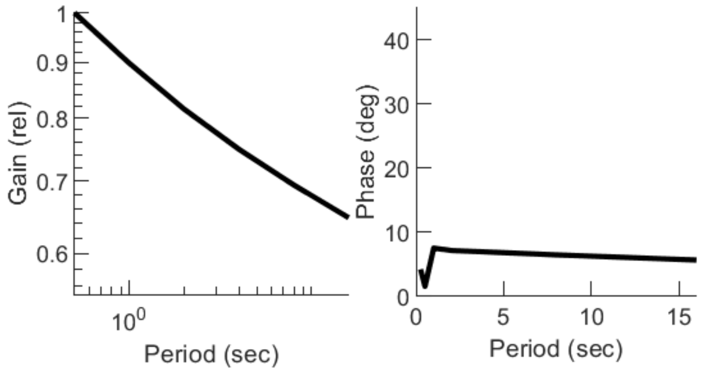
- Fractional order differentiation has been shown in
  - Cortical
    - Lundstrom, B. N., Higgs, M. H., Spain, W. J., & Fairhall, A. L. (2008). Fractional differentiation by neocortical pyramidal neurons. *Nature neuroscience*, *11*(11), 1335.
  - Brainstem
    - Anastasio, T. J. (1994). The fractional-order dynamics of brainstem vestibulo-oculomotor neurons. *Biological cybernetics*, *72*(1), 69-79.
  - Weakly electric fish.
    - Huang, C. G., & Chacron, M. J. (2016). Optimized parallel coding of second-order stimulus features by heterogeneous neural populations. *Journal of Neuroscience*, *36*(38), 9859-9872.
  - Insects
    - French, A. S. (1984). Dynamic properties of the action potential encoder in an insect mechanosensory neuron. *Biophysical journal*, *46*(2), 285-289.

Our preliminary data on Purkinje cells show these cells perform a fractional order differentiation



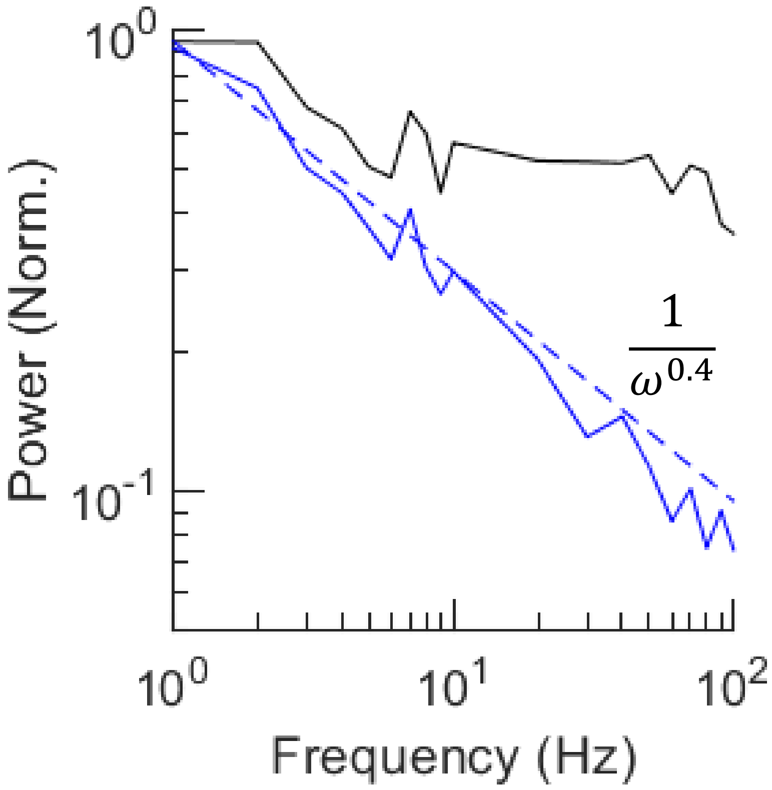
Fractional differentiation and signal whitening by the fLIF

Fractional differentiation



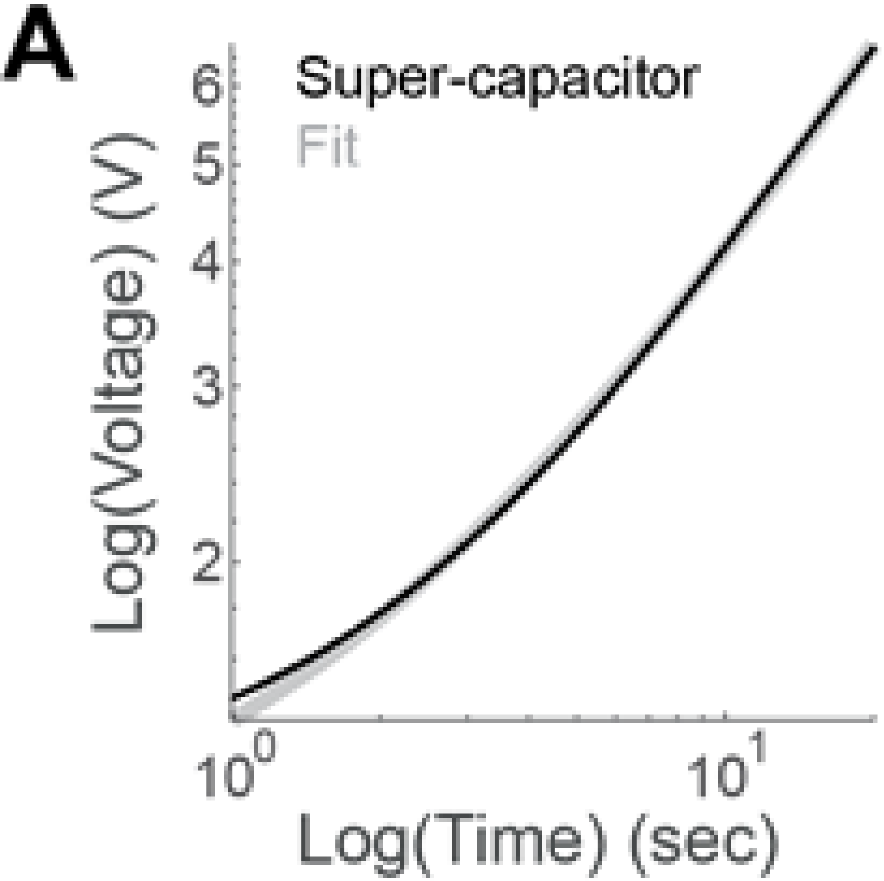
$$(2\pi\omega)^{\eta}\sin\left(2\pi\omega t+\frac{\eta\pi}{2}\right)$$

Noise whitening



Instantiating the circuits

A super capacitor is a fractional order differentiator



Vazquez-Guerrero, Tuladhar et al. (2024) Scientific Reports

Importance for AI

- Electric elements with memory are known as memelements
  - Memristors, memcapacitors, meminductors
  - Memristor-The missing circuit element, L. Chua, IEE Trans on circuit theory, 1971
- Intense interest
  - Implement neuronal functions intrinsically in hardware
  - Lower energy consumption
- **Most people care about memristors**
  - **Materials**
  - **Used to model synapses**
- **Capacitors can me many times more energetically efficient than resistors**

Real capacitors as memcapacitors

- A memcapacitor is

$$q = C\left(x, v, t\right)v \text{ and } D^1 x = f(x, v, t)$$

- Where q is charge, C is capacitance, and x is an internal variable such as the flux:

$$D^1 \varphi = v$$

Assuming that $C\left(\varphi, v, t\right) = C_0 \varphi$ then

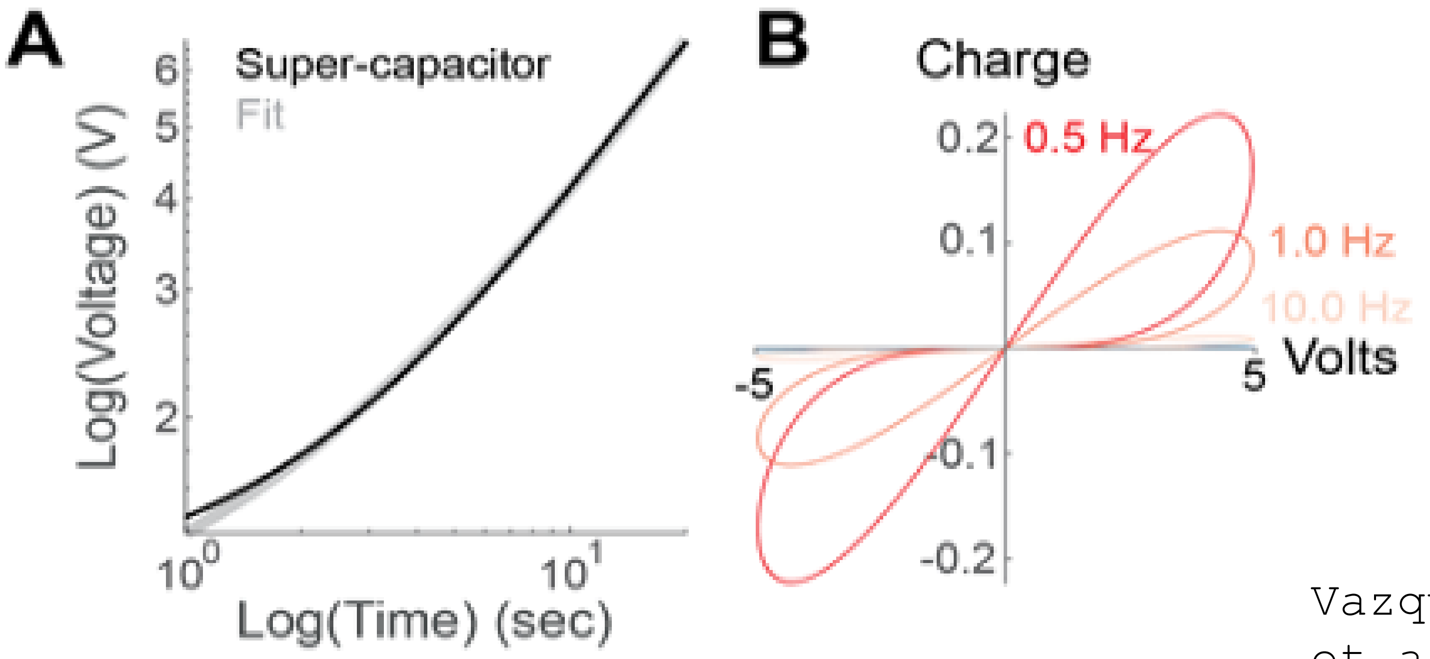
$$\mathrm{q} = C_0 \int_0^t v d\tau \cdot v$$

Check out any Chua publication for details

This is a hysteresis process pinched at the center

Fractional order capacitors and memcapacitors

- A fractional order differentiator and a memcapacitor



Vazquez-Guerrero, Tuladhar et al. (2024) Scientific Reports

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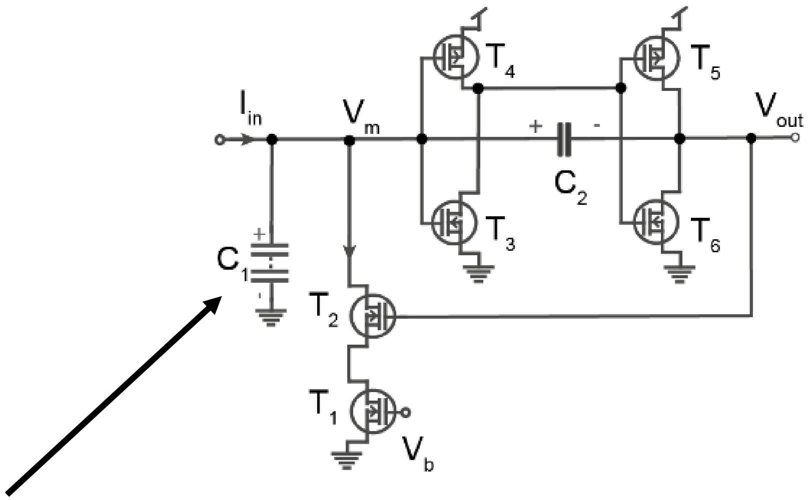
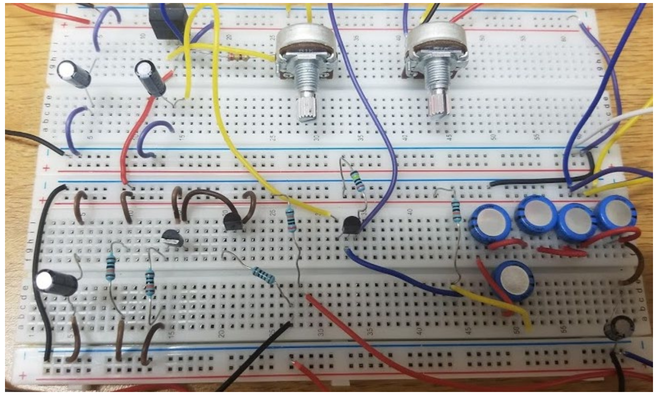


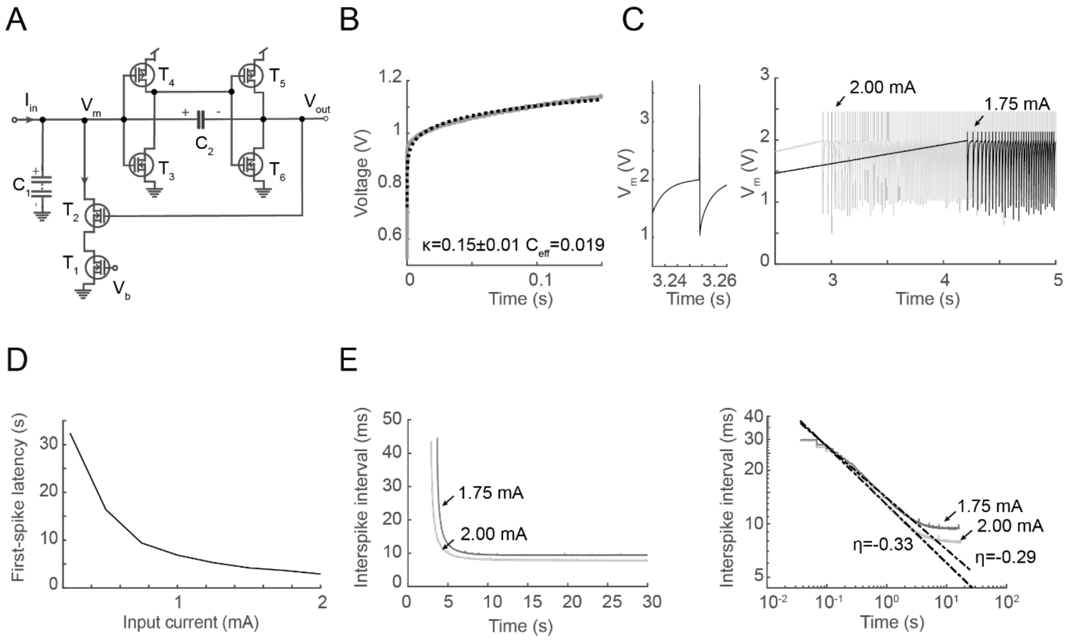


Super-capacitor

Based on the classic design by Carver Mead

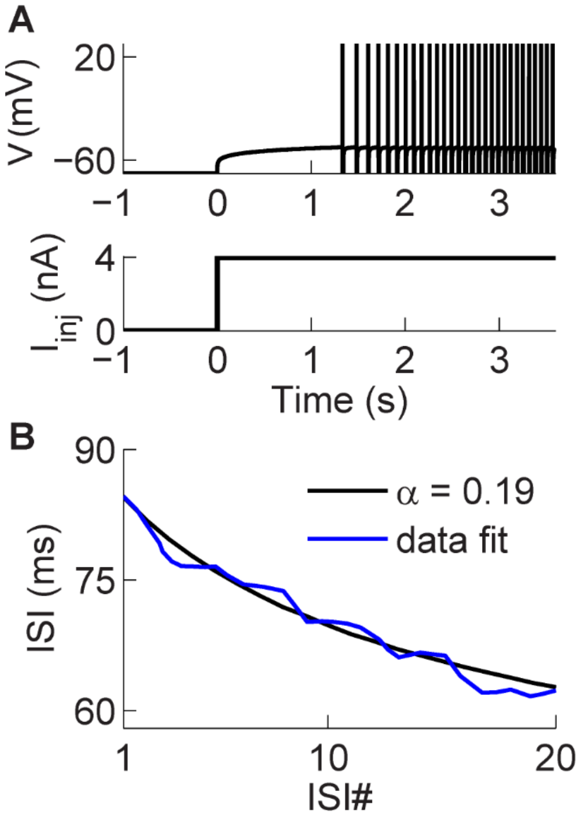
The fLIF circuit vs model vs data

Fractional circuit

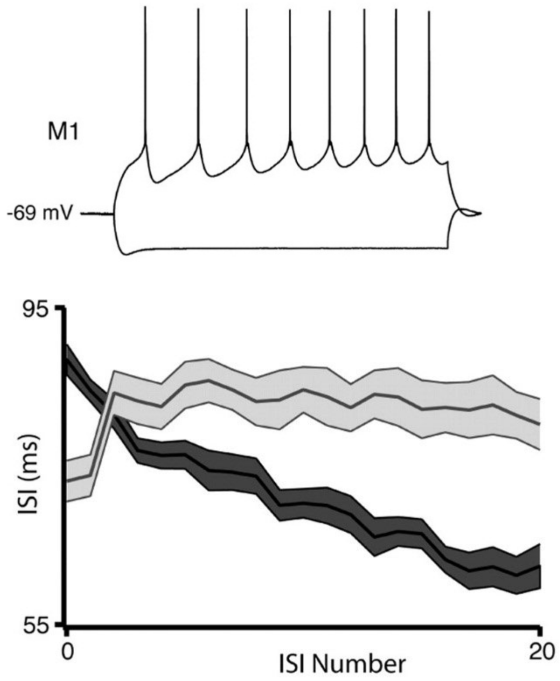


Vazquez-Guerrero, Tuladhar et al. (2024) Scientific Reports

Fractional model



Experiments



The fLIF circuit vs model vs data

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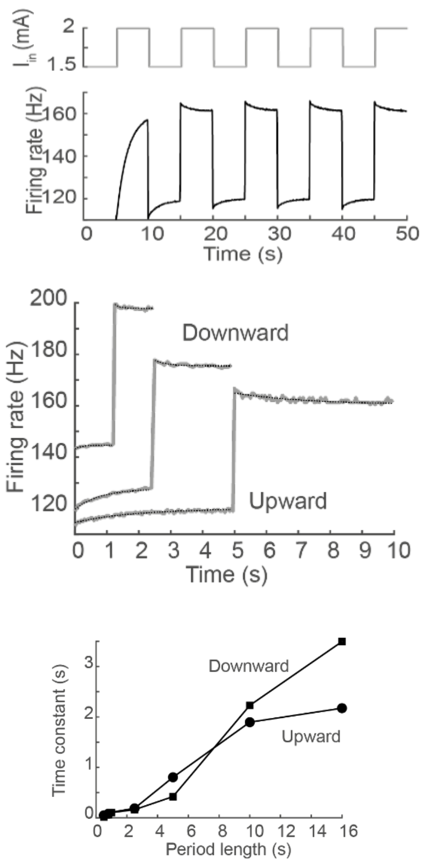

Vazquez-Guerrero, Tuladhar et al. (2024) Scientific Reports

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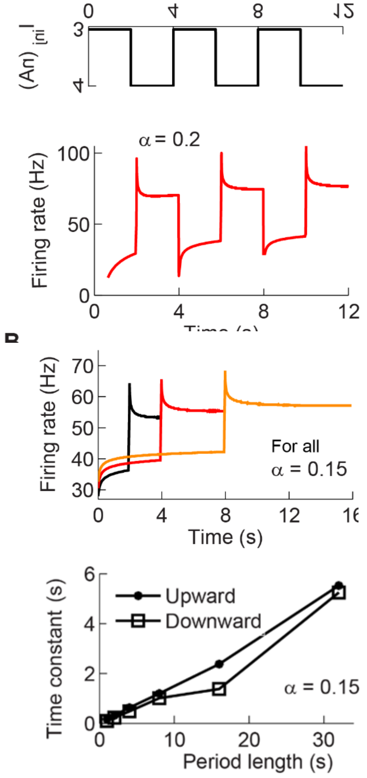
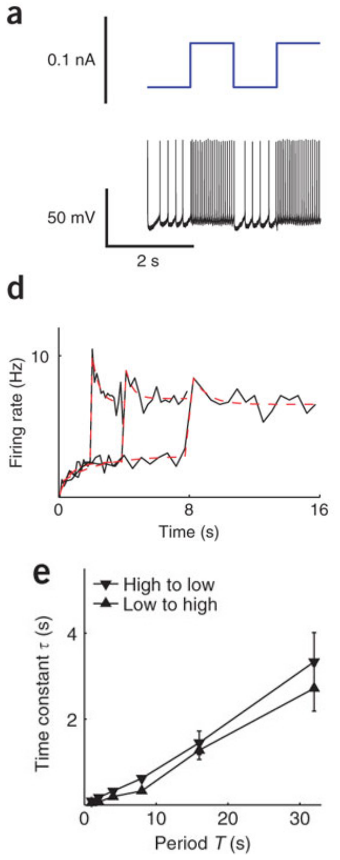


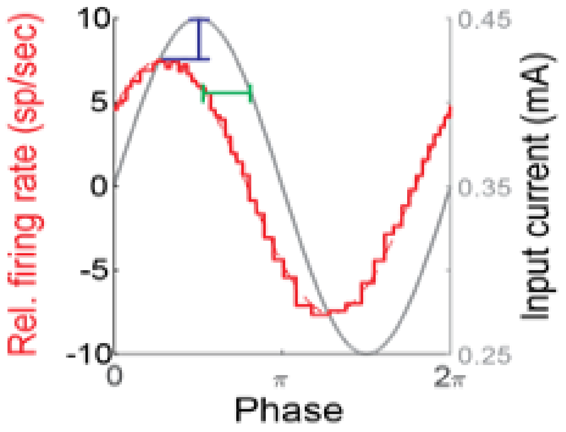
Teka et al., PLoS Comp. Bio 2014.

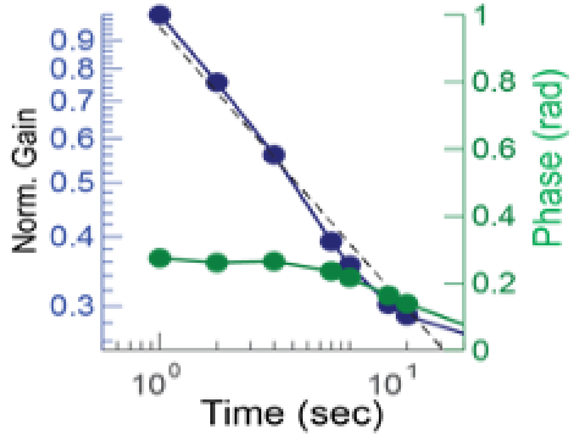
Experiments



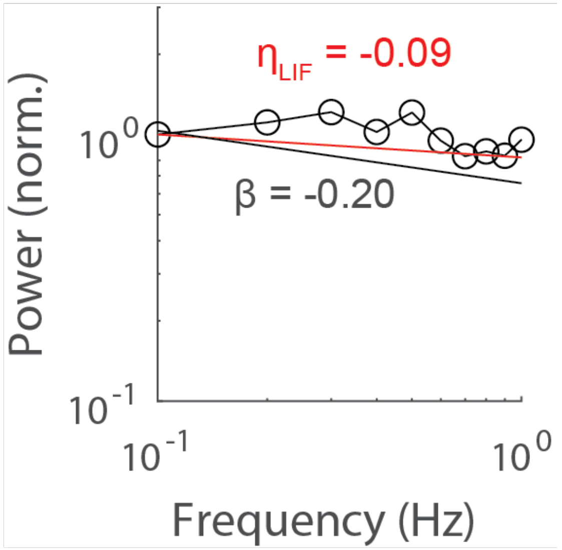
Lundstrom et al., Nat. Neurosci. 2008

Fractional differentiation and input whitening





$$f = \frac{d^{\eta} I_n}{dt^{\eta}} \rightarrow f = A(2\pi\omega)^{\eta} \sin\left(2\pi\omega t + \frac{\eta\pi}{2}\right)$$



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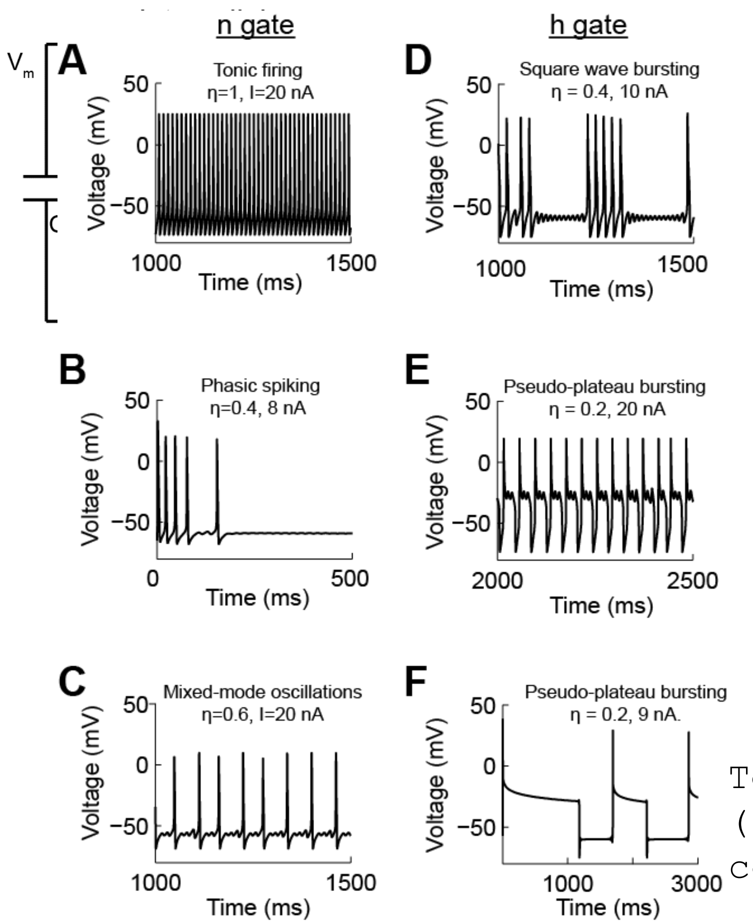
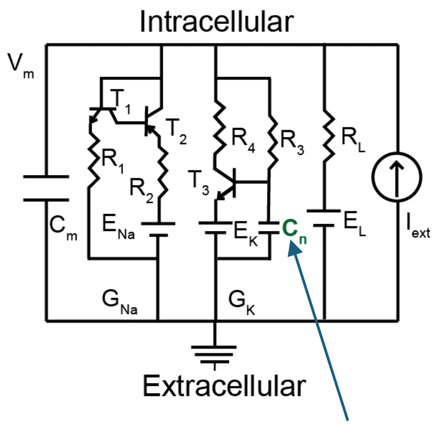
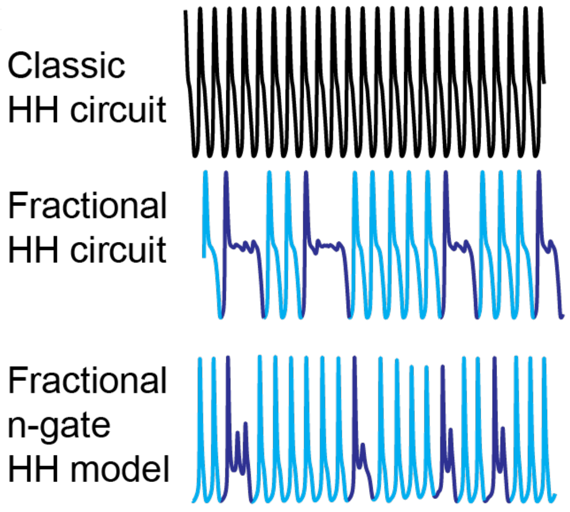
A fractional order Hodgkin-Huxley circuit



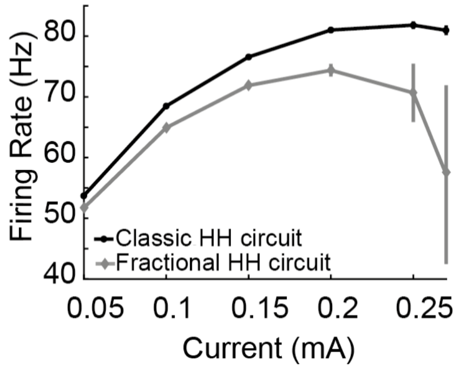
Teka, Stockton et al. (2016) PLoS computational biology



Super-capacitor stack 1 mF

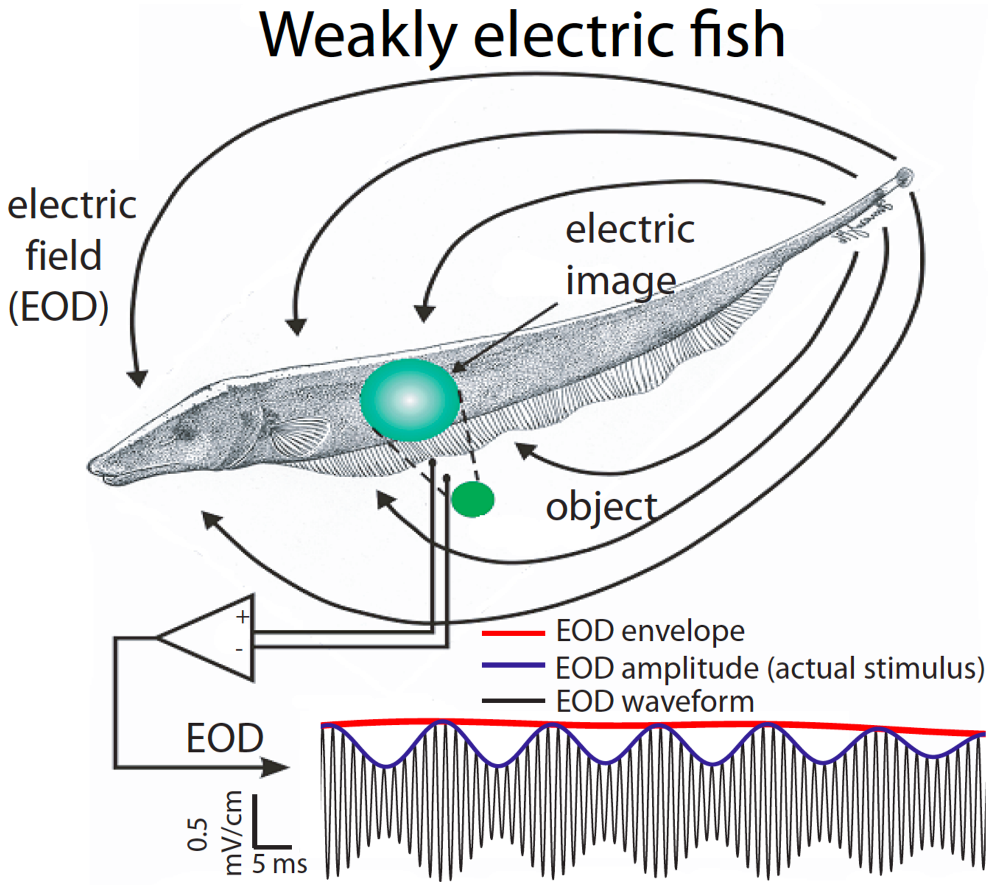


Vazquez-Guerrero, Tuladhar et al. (2024) Scientific Reports



Compare the circuit to experiments

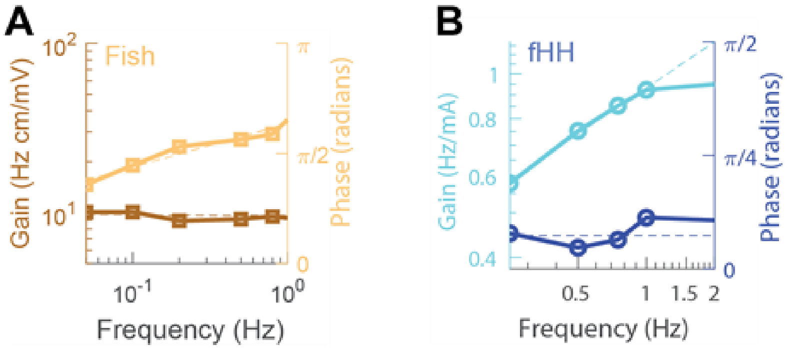
- The pyramidal cells of the ELL perform a fractional derivative of the sensory input.
- Fractional dynamics of these neurons controlled by a potassium conductance
- Recording multiple neuron using neuropixel probes.
- Collaboration with Maurice Chacron @ McGill



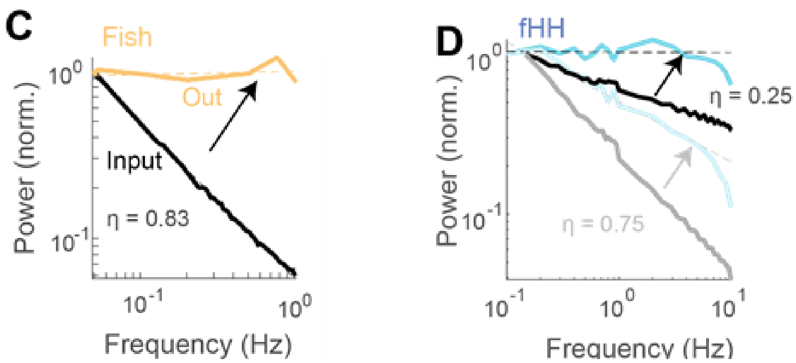
Huang, C. G., & Chacron, M. J. (2016). Optimized parallel coding of second-order stimulus features by heterogeneous neural populations. *Journal of Neuroscience*, *36*(38), 9859-9872.

The HH model and real data from weakly electric fish

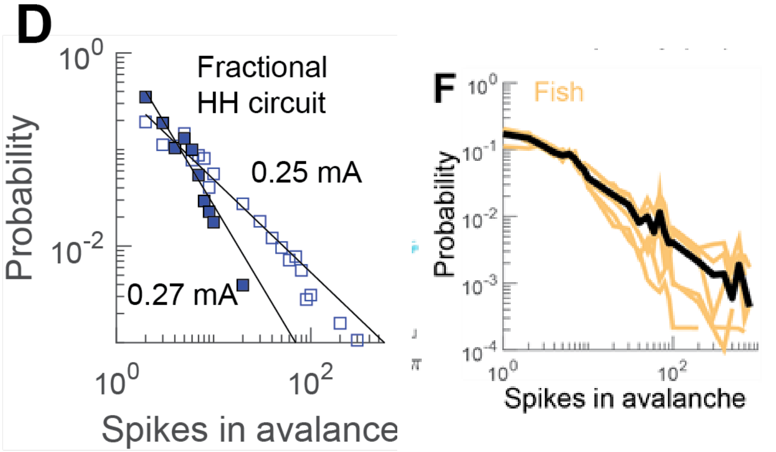
Fractional differentiation



Input whitening



Criticality



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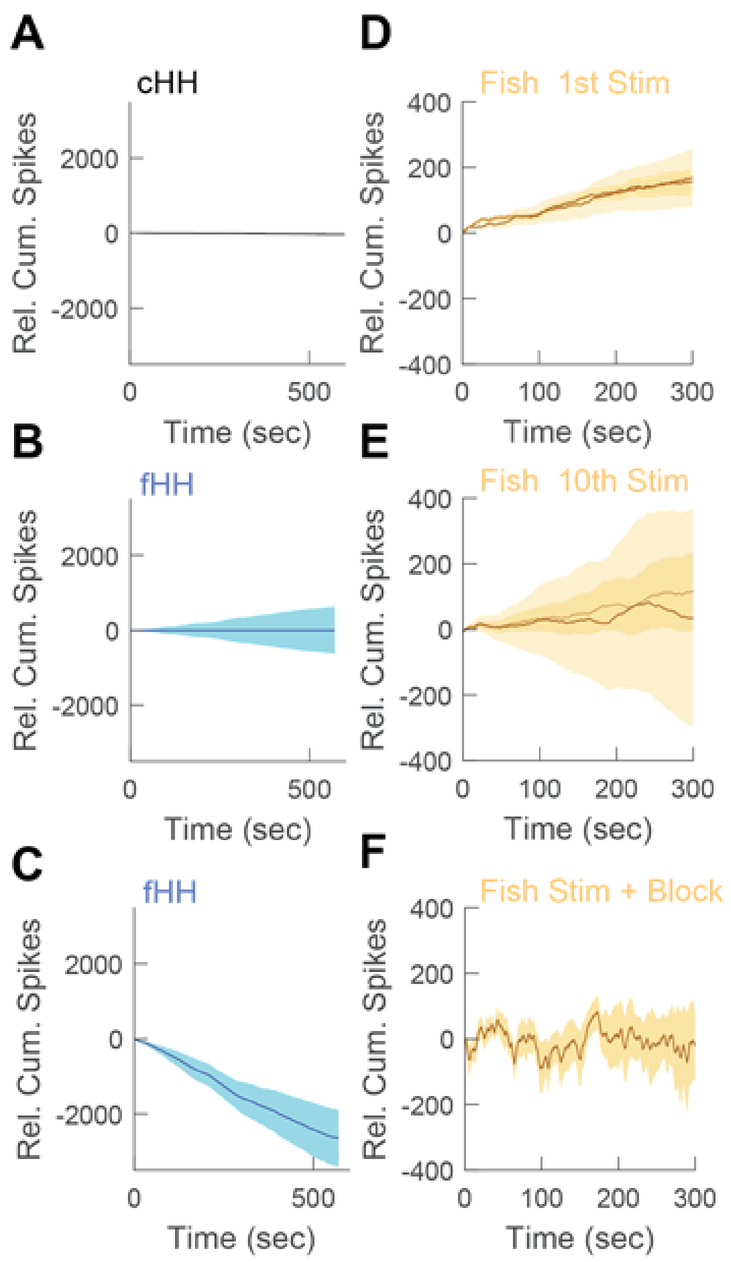
Predicting neuronal responses in live fish



Vazquez-Guerrero, Tuladhar et al. (2024) Scientific Reports

Towards fractional dynamics neuromorphic engineering

- Complexity in time at single cell scale provides optimal coding usually attributed to networks
  - Neurons adapt!
  - There is more to computation than synaptic plasticity
  - Intrinsic excitability should be incorporated into neuromorphic studies and designs
- Fractional order memcapacitor networks could provide
  - Optimal encoding – natural and human made signals
  - Optimal energy – the capacitor is the most efficient electric element
- Fractional differentiation in neurons exists at multiple scales
  - It can be measured
  - It is not a metaphor exported to AI

Collaborators

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Unified theory of adaptation

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Fractional-order neuronal dynamics

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Neuro-Inspired AI for the Edge

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End